Basics of Tao web and its application to anomaly polynomial

Masato Taki RIKEN, iTHES

work in progress

Collaboration w/ S-S.Kim and F.Yagi PTEP 2015,083B02

+H.Hayashi and K.Lee JHEP 1508:097 2015

etc

2016.12/16
@KIAS

What I want to discuss is…

New brane description of 6d SCFTS: tao web

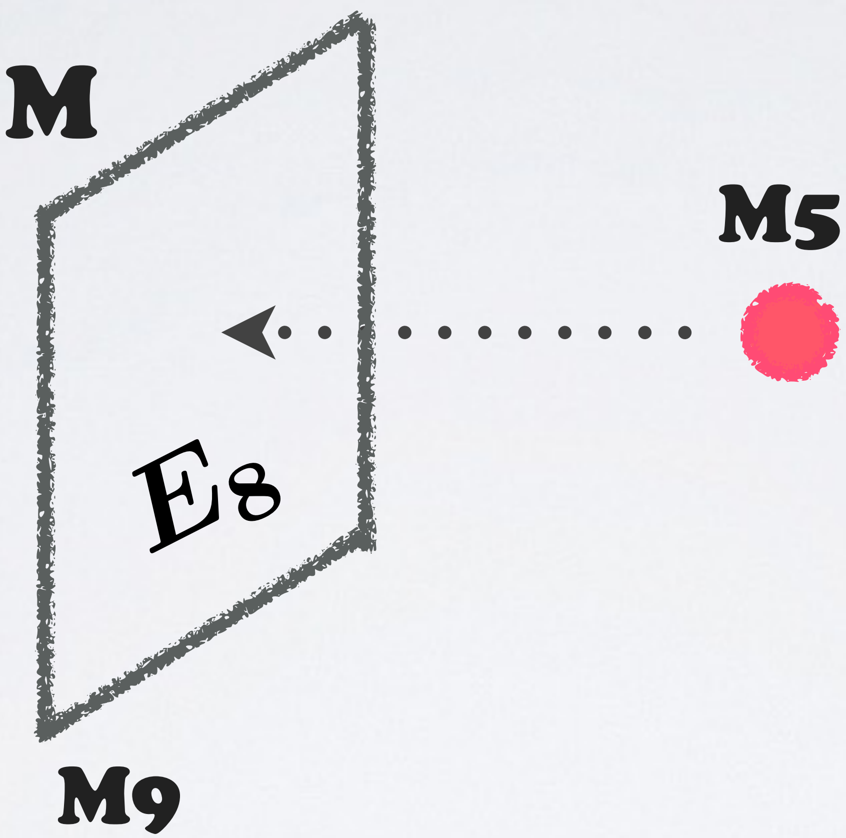
w/ S-S.Kim and F.Yagi etc

It's application of computation of physical quantities(eg. anomaly)

E-string theory and web

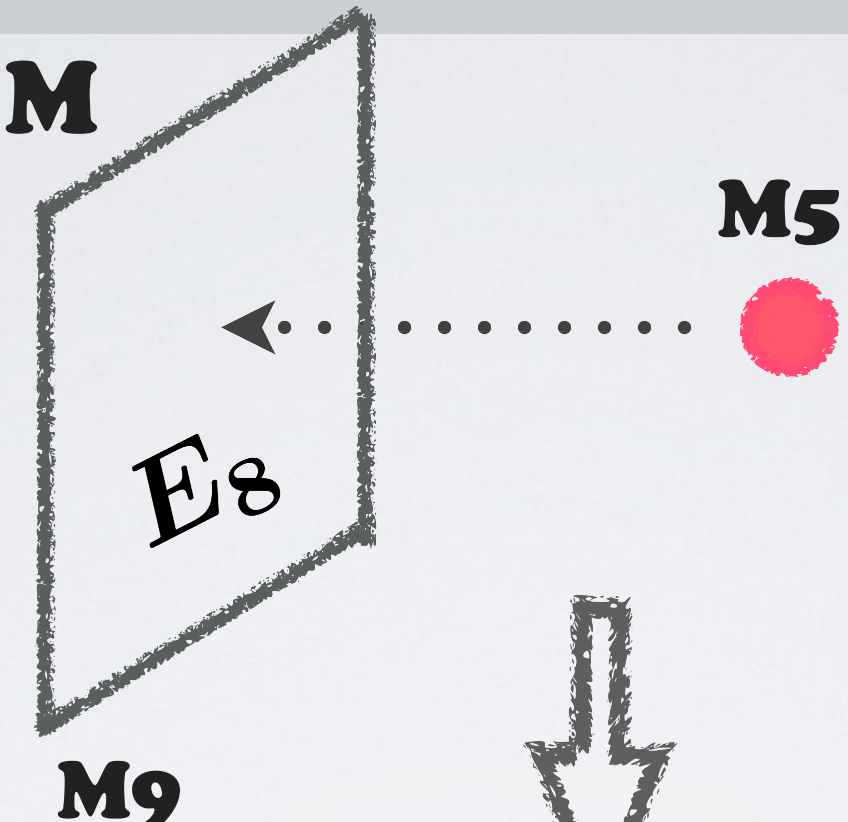
6d E-string theory

* Heterotic M

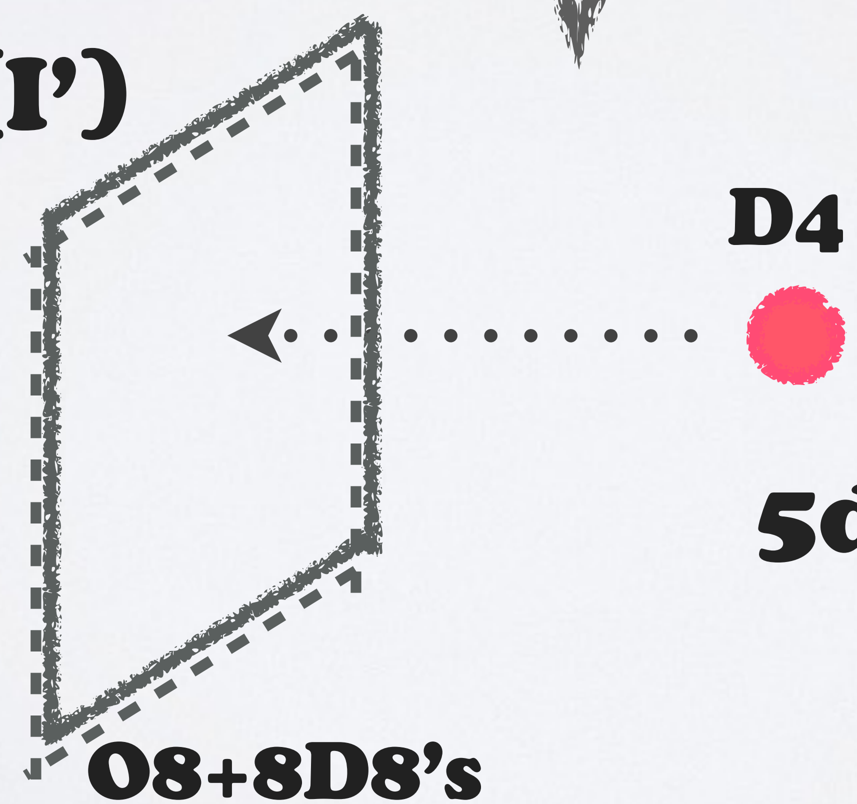


6d E-string theory

* Heterotic M



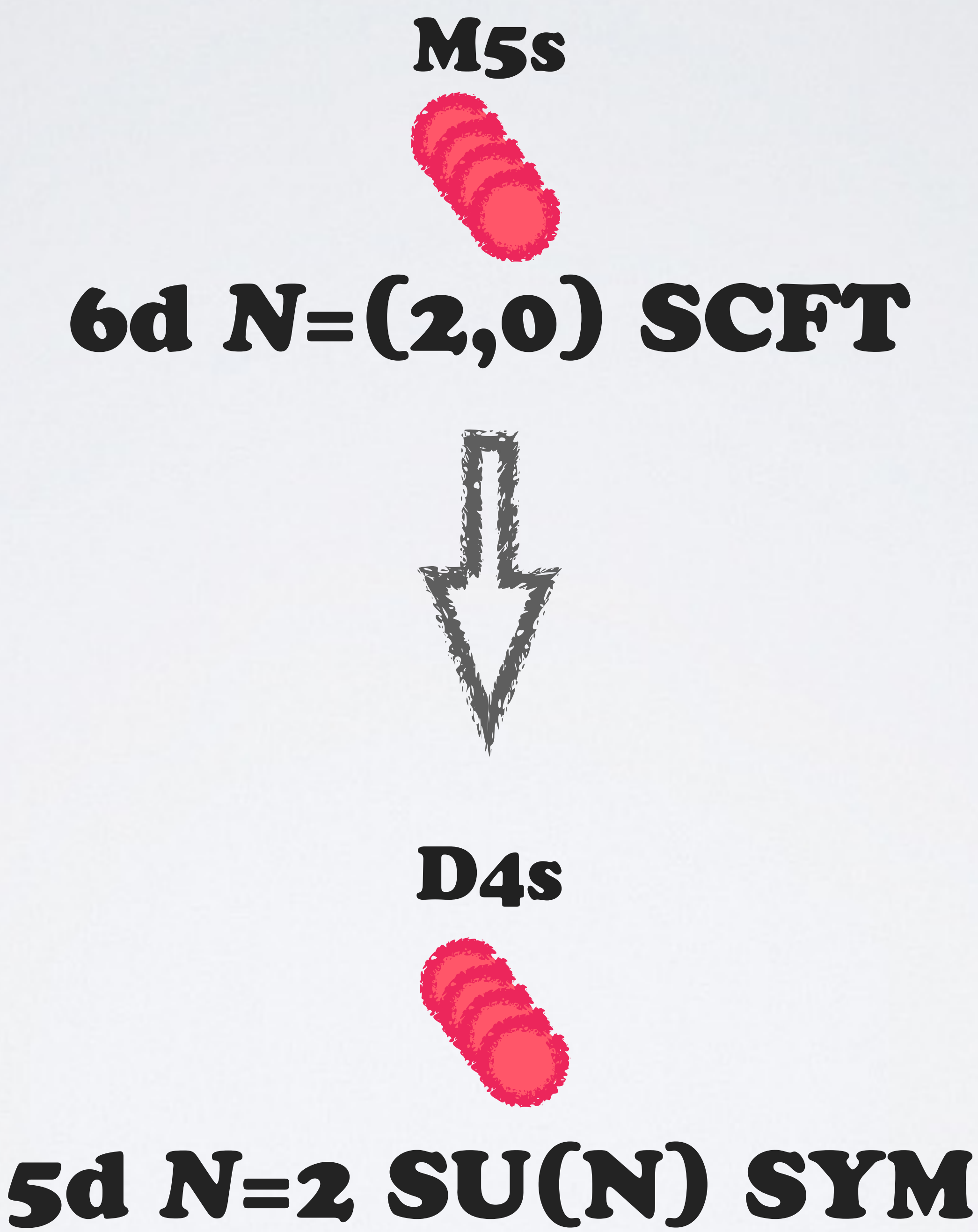
* Type IIA (I')



5d Sp(1) 8 flavors

Analog to 6d N=(2,0) case

* M-theory
* Type IIA



Our starting point

6d E-string theory is UV fixed point of the 5d theory:

$\mathcal{N}=1$ SU(2) $N_f=8$ SQCD

* $N_f<8$ gives 5d fixed point

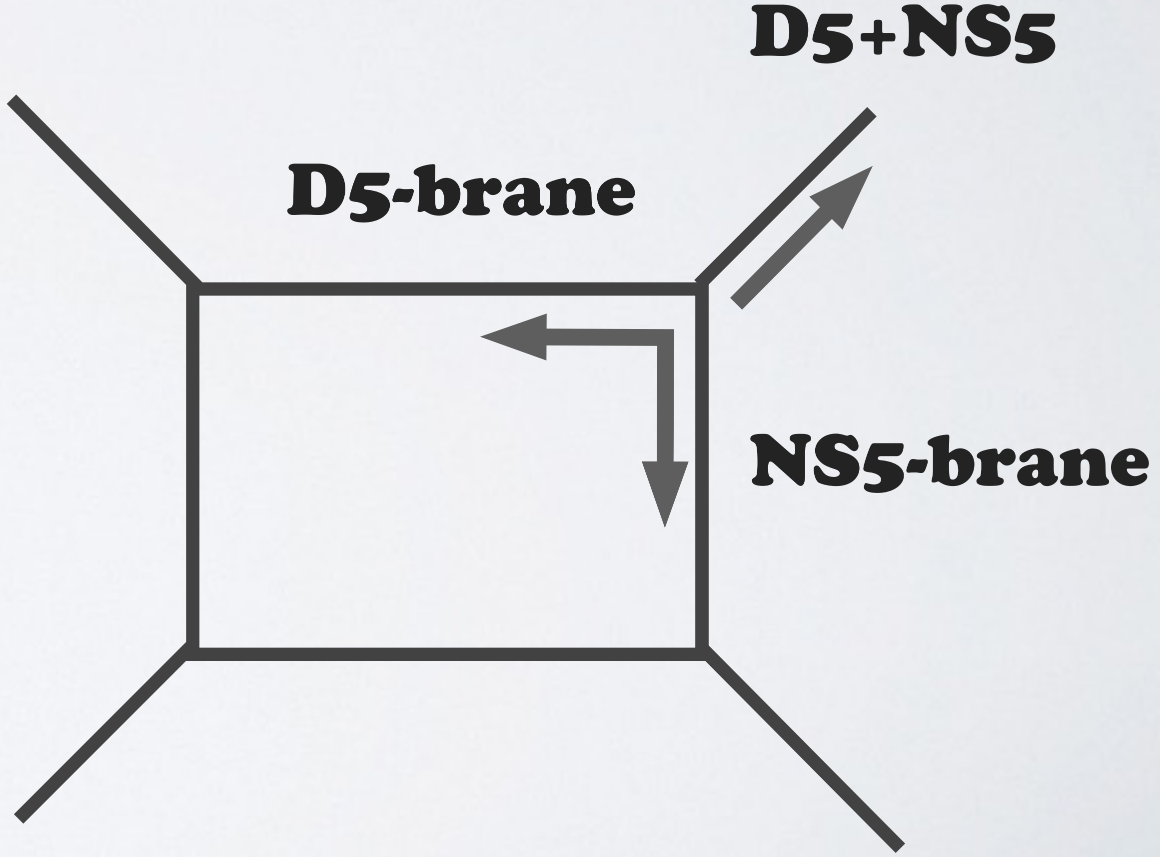
N_f<8 SU(2) theories

5d $\mathcal{N}$=1 theory : engineered by 5-brane web

N_f<8 SU(2) theories

5d $\mathcal{N}$=1 theory : engineered by 5-brane web

eg) SU(2) Pure Super Yang-Mills



N_f<8 SU(2) theories

5d $\mathcal{N}$=1 theory : engineered by 5-brane web

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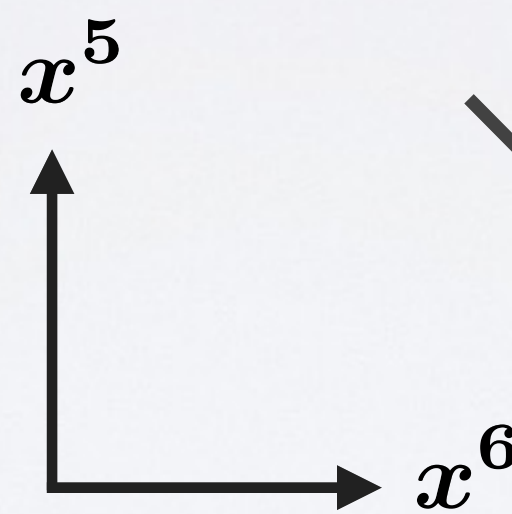
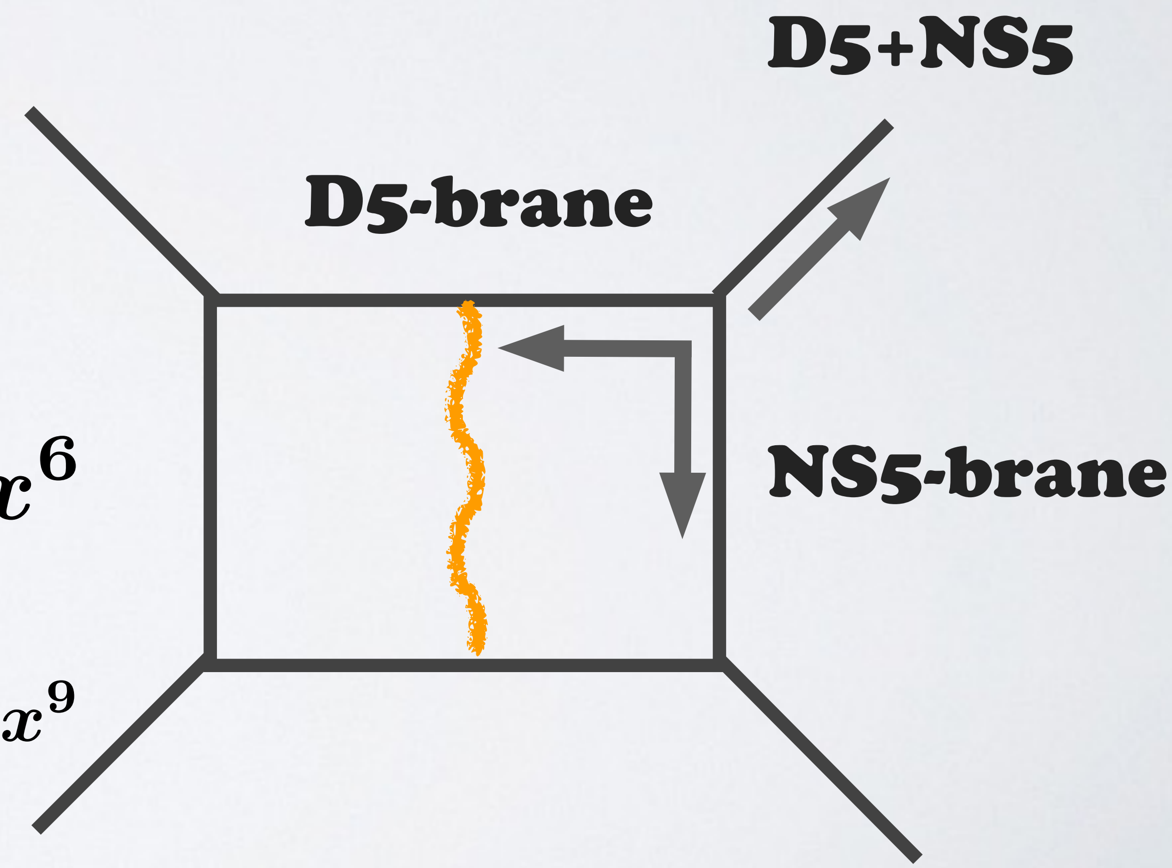


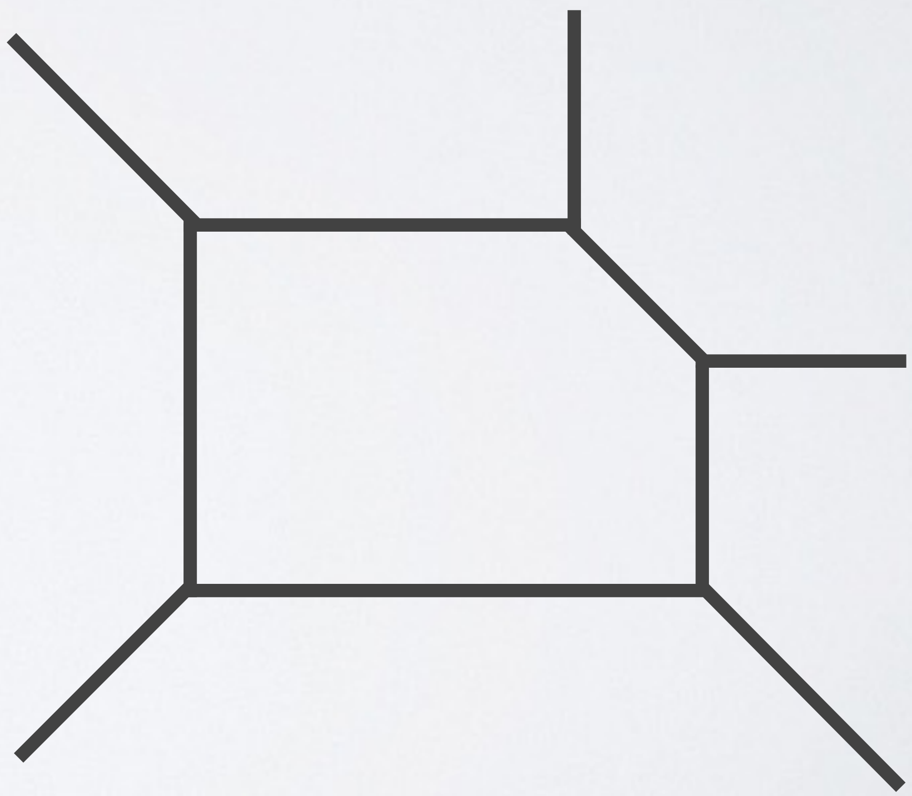


	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
5-brane	○	○	○	○	○	1d confg				

N_f<8 SU(2) theories

5d $\mathcal{N}$=1 theory : engineered by 5-brane web

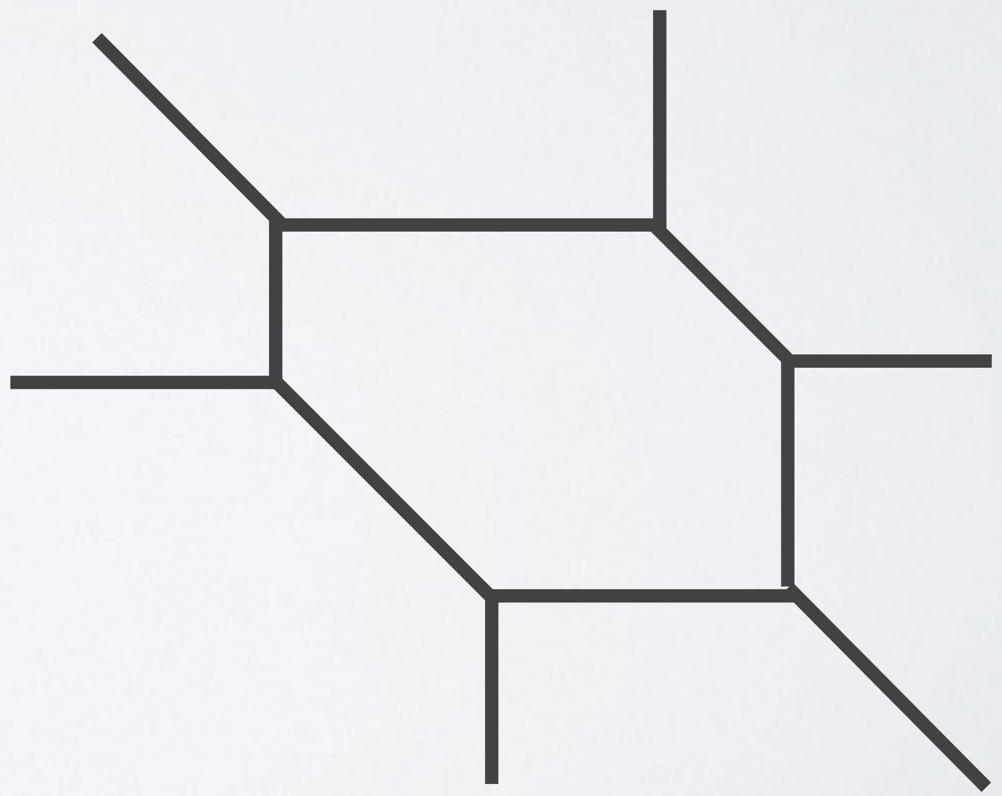
eg) SU(2) N_f=1 SQCD



N_f<8 SU(2) theories

5d $\mathcal{N}$=1 theory : engineered by 5-brane web

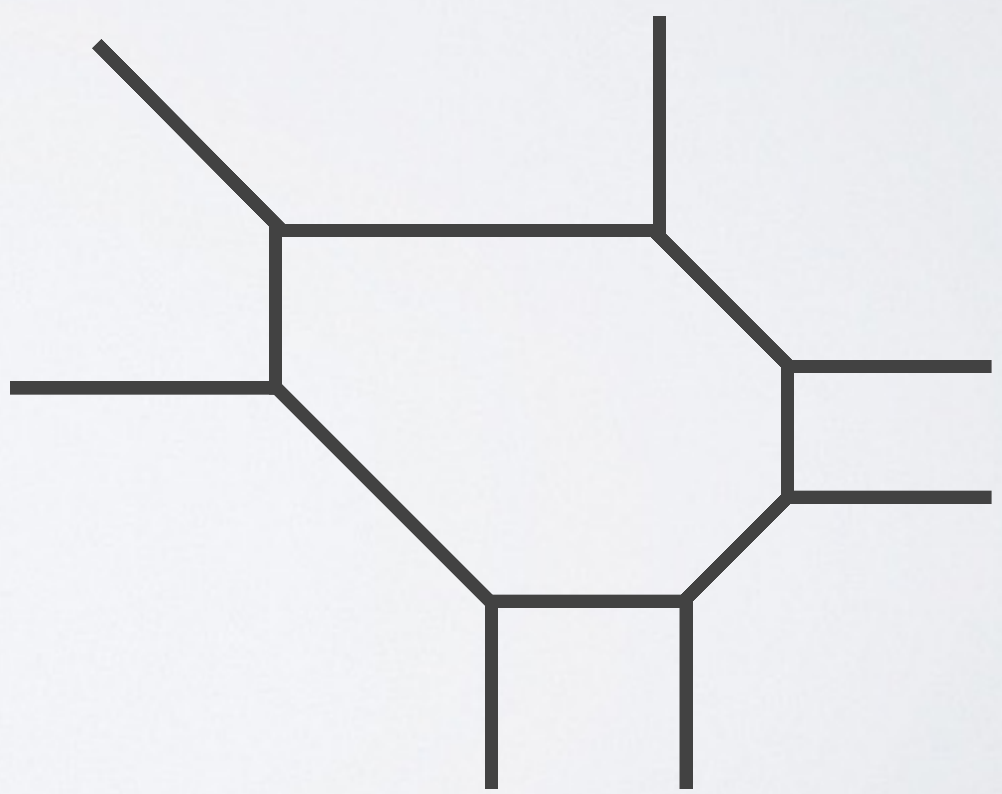
eg) SU(2) N_f=2 SQCD



N_f<8 SU(2) theories

5d $\mathcal{N}$=1 theory : engineered by 5-brane web

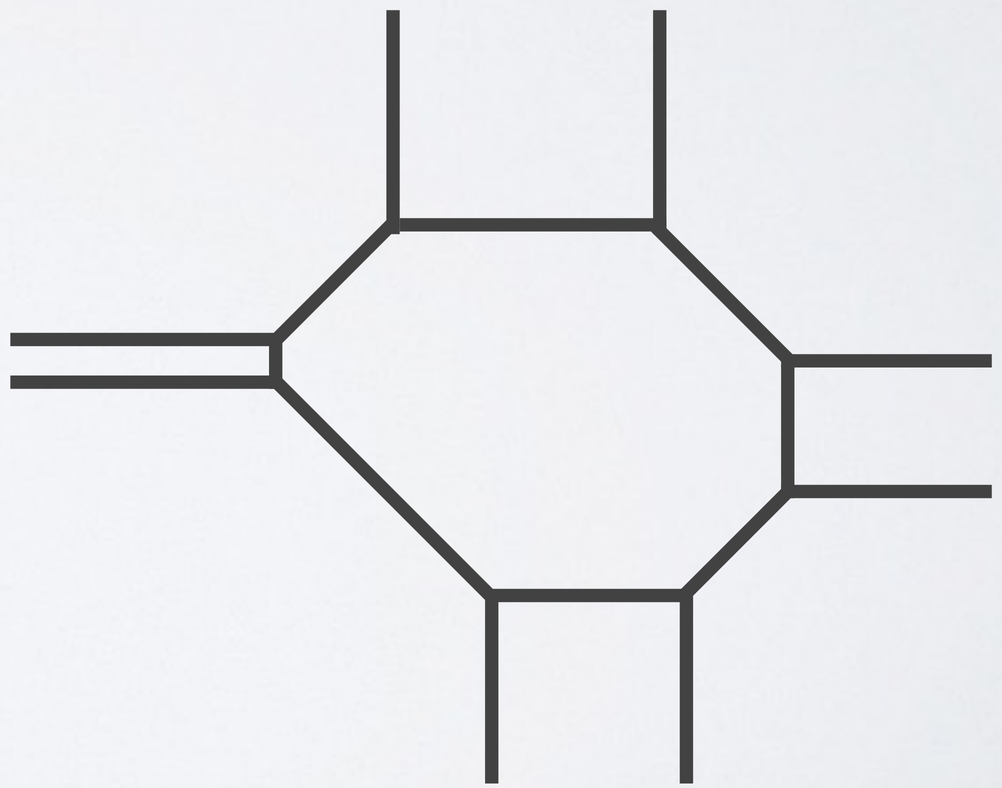
eg) SU(2) Nf=3 SQCD



N_f<8 SU(2) theories

5d $\mathcal{N}$=1 theory : engineered by 5-brane web

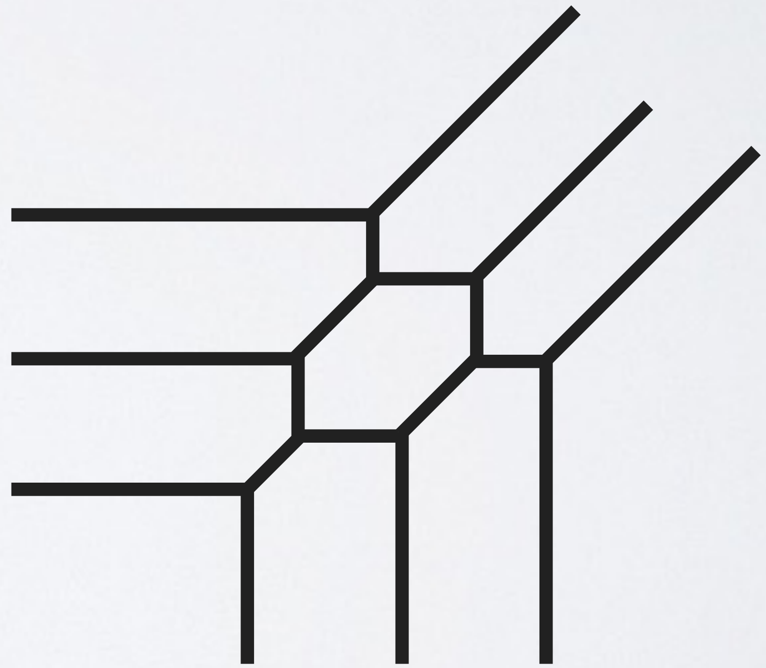
eg) SU(2) N_f=4 SQCD



N_f<8 SU(2) theories

5d $\mathcal{N}$=1 theory : engineered by 5-brane web

eg) SU(2) Nf=5 SQCD

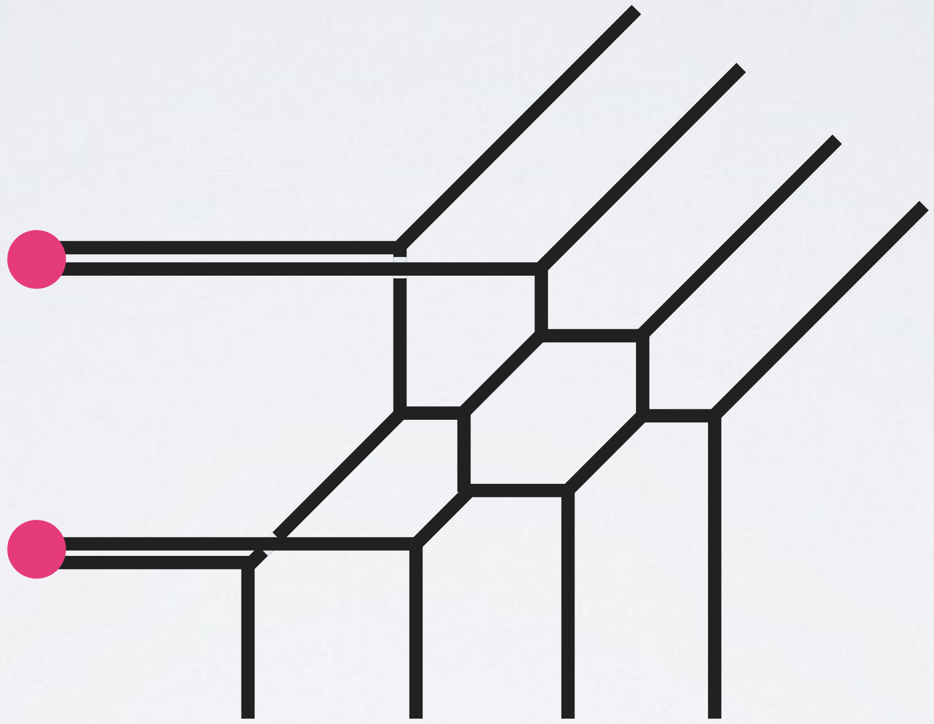


we can go beyond N_f=5 !

[Benini-Benvenuti-Tachikawa, ’09]

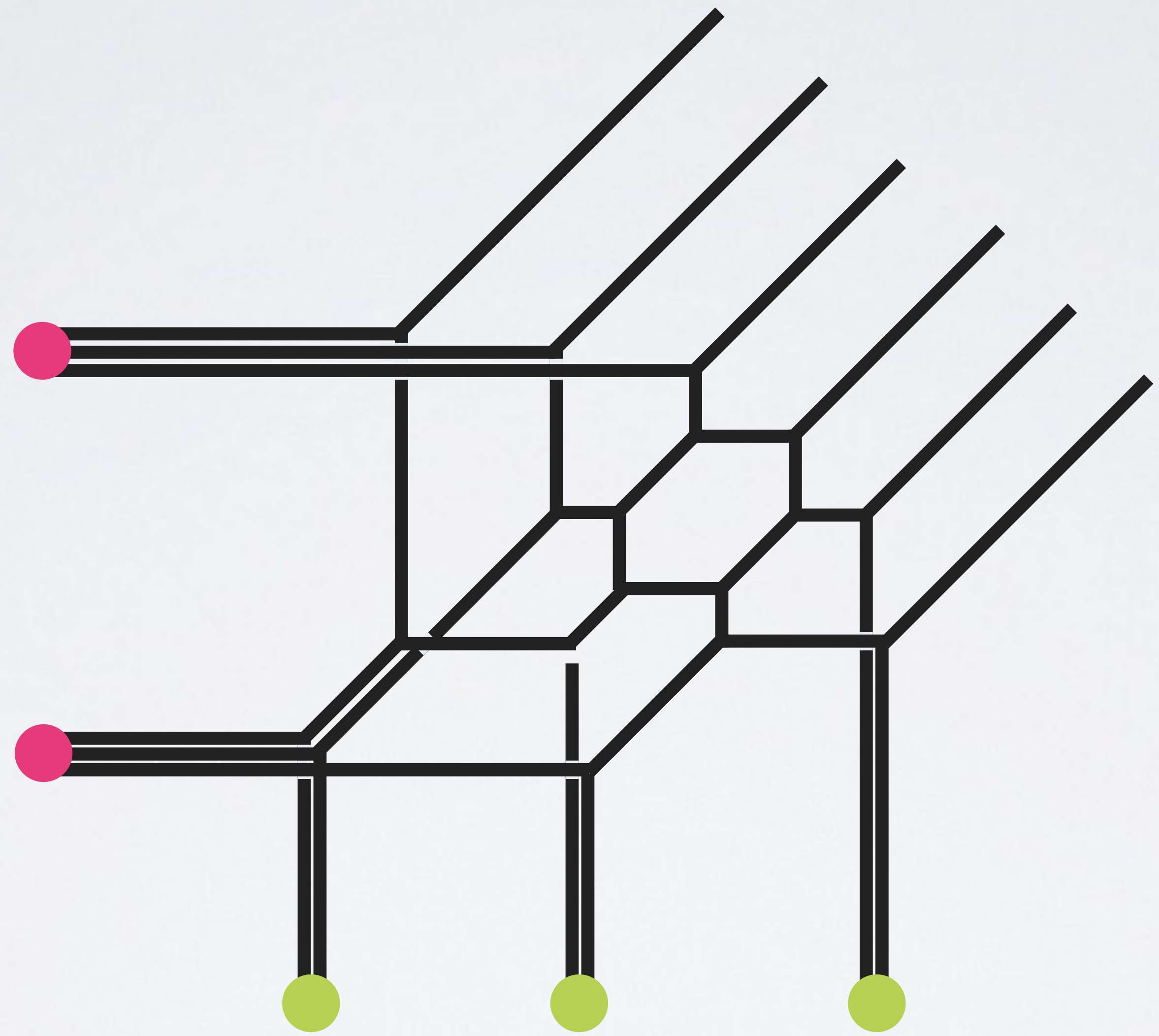
Web for N_f=6 theory

[Benini-Benvenuti-Tachikawa, '09]



Web for N_f=7 theory

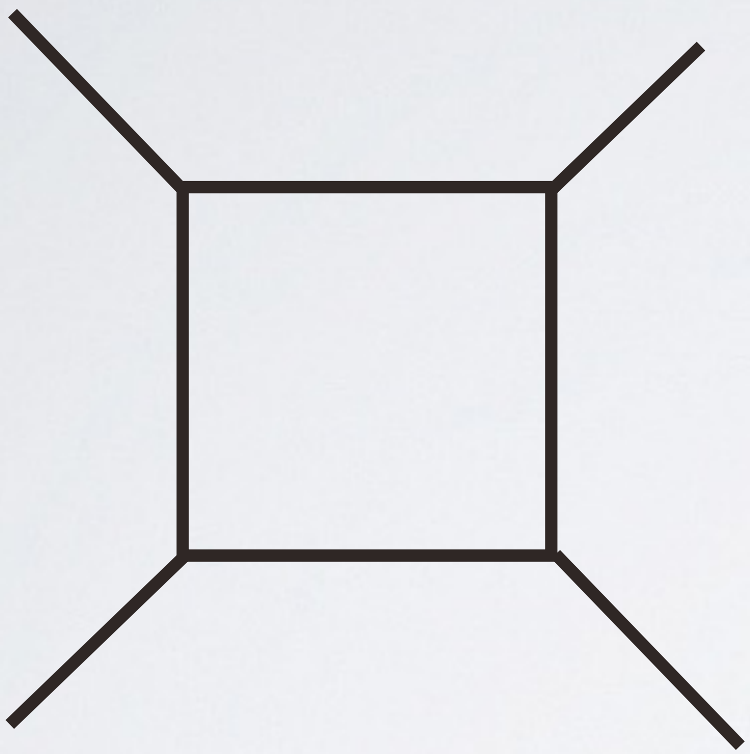
[Benini-Benvenuti-Tachikawa, ’09]

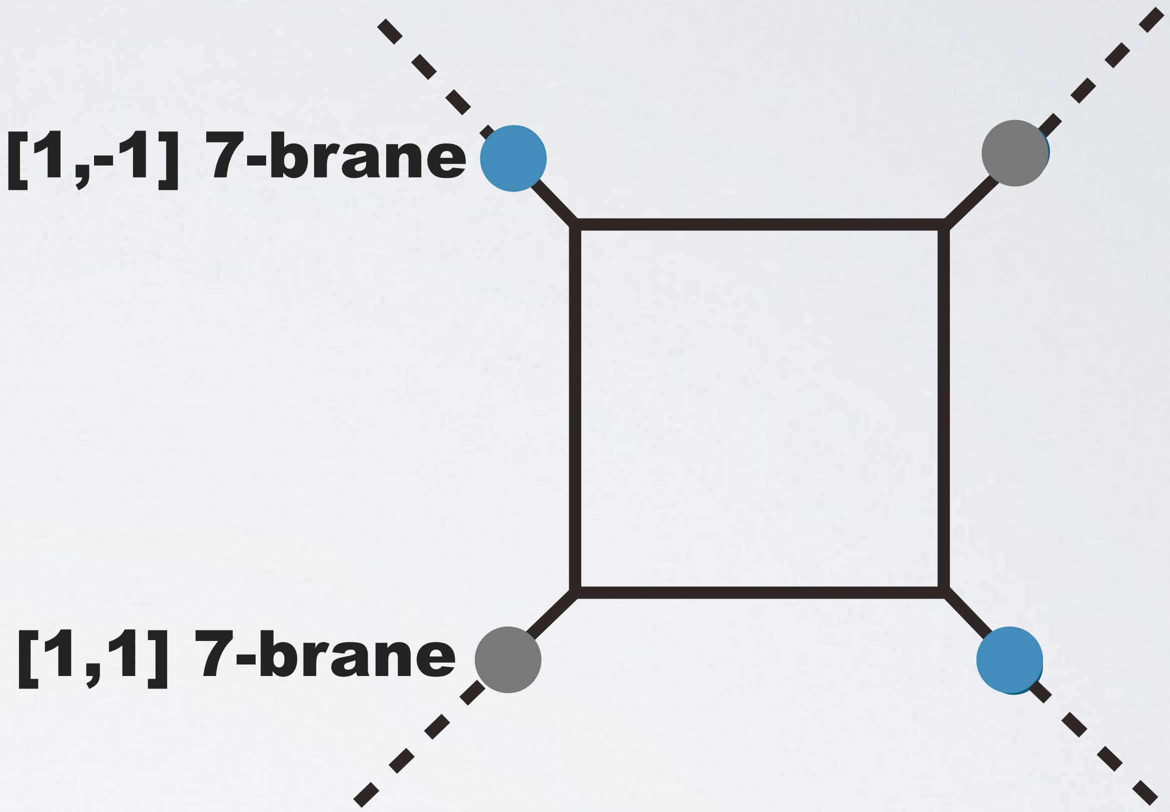


Systematic Method: 7-brane technology

5d N=1 SU(2) pure YM

[DeWolfe-Hanany-Iqbal-Katz, '99]





5d N=1 SU(2) pure YM

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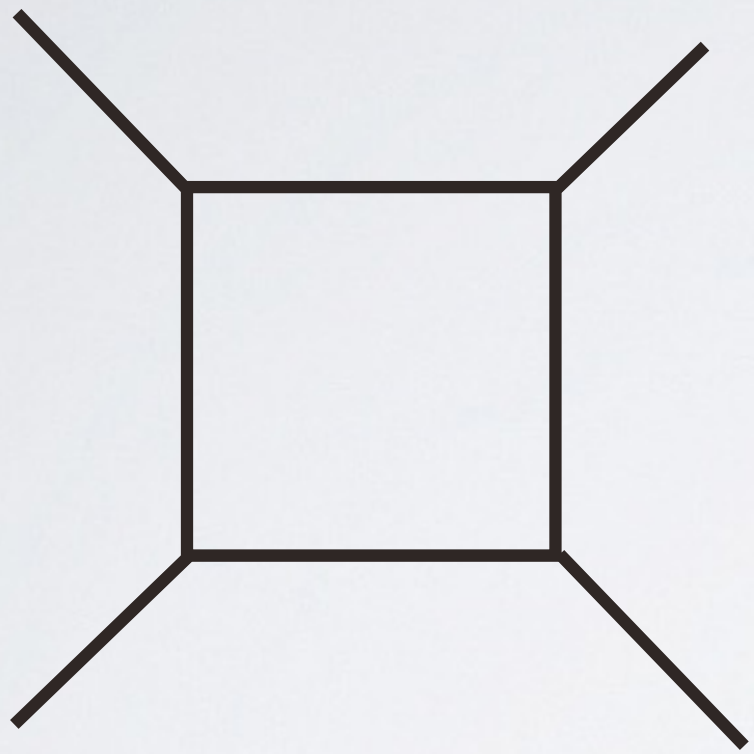
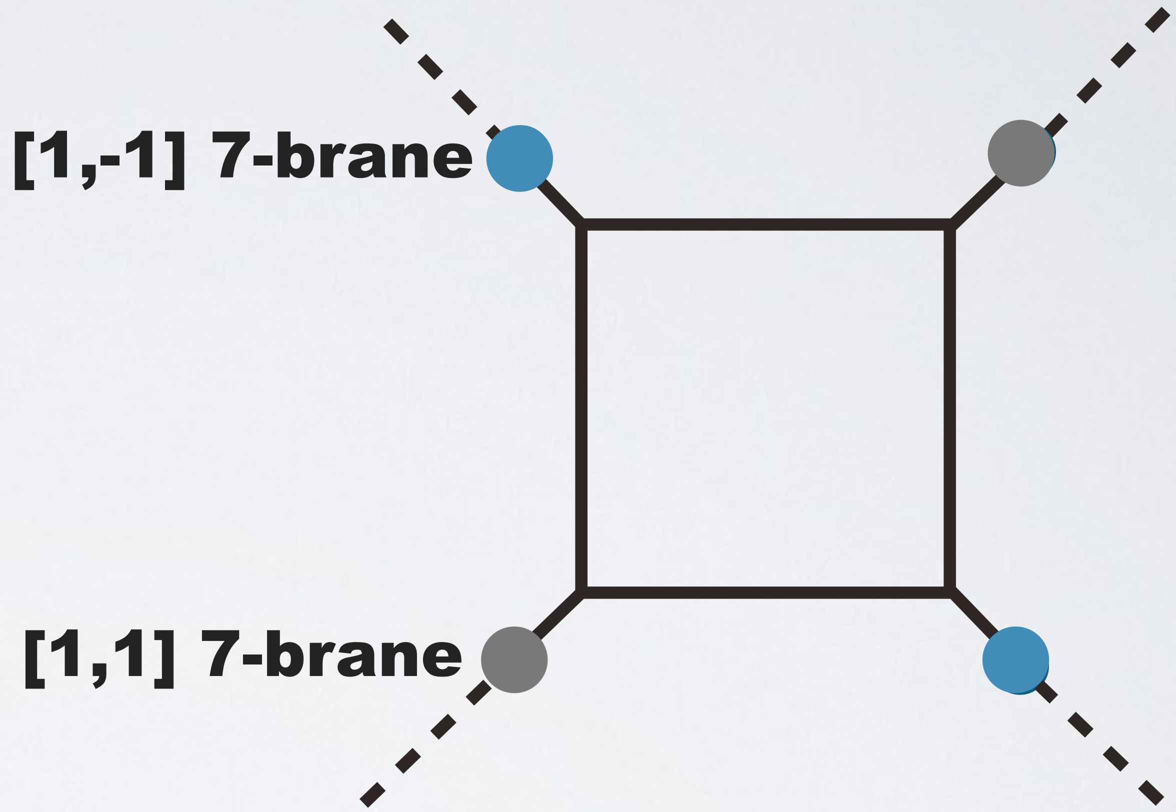
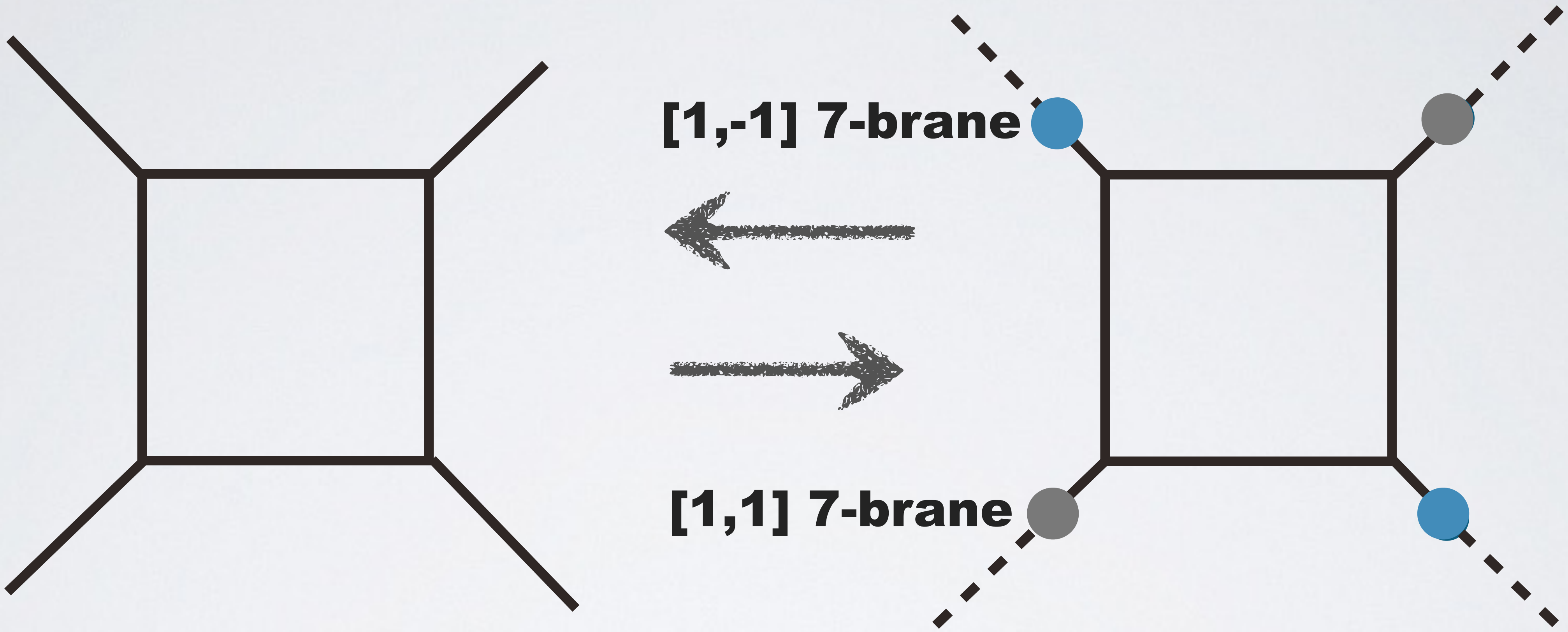




	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
5-brane	○	○	○	○	○	1d confg				
7-brane	○	○	○	○	○	pt		○	○	○

5d N=1 SU(2) pure YM

[DeWolfe-Hanany-Iqbal-Katz, '99]



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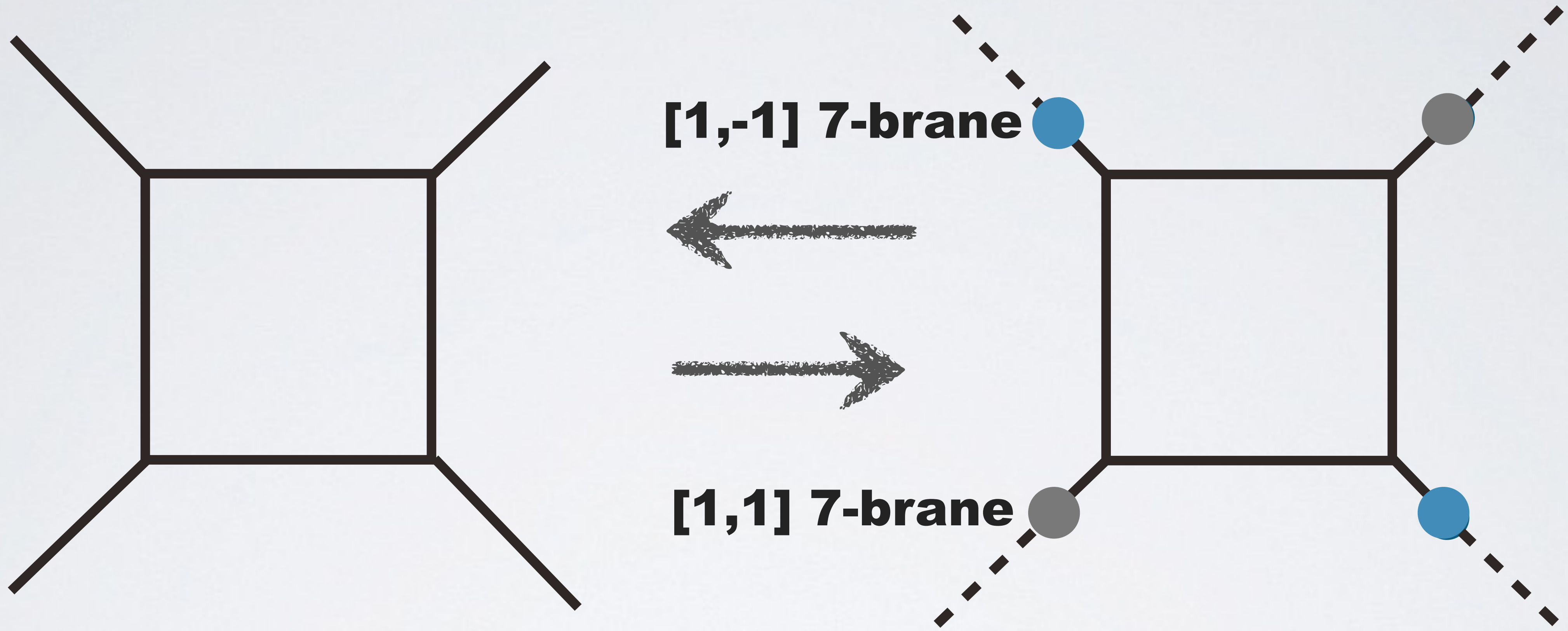
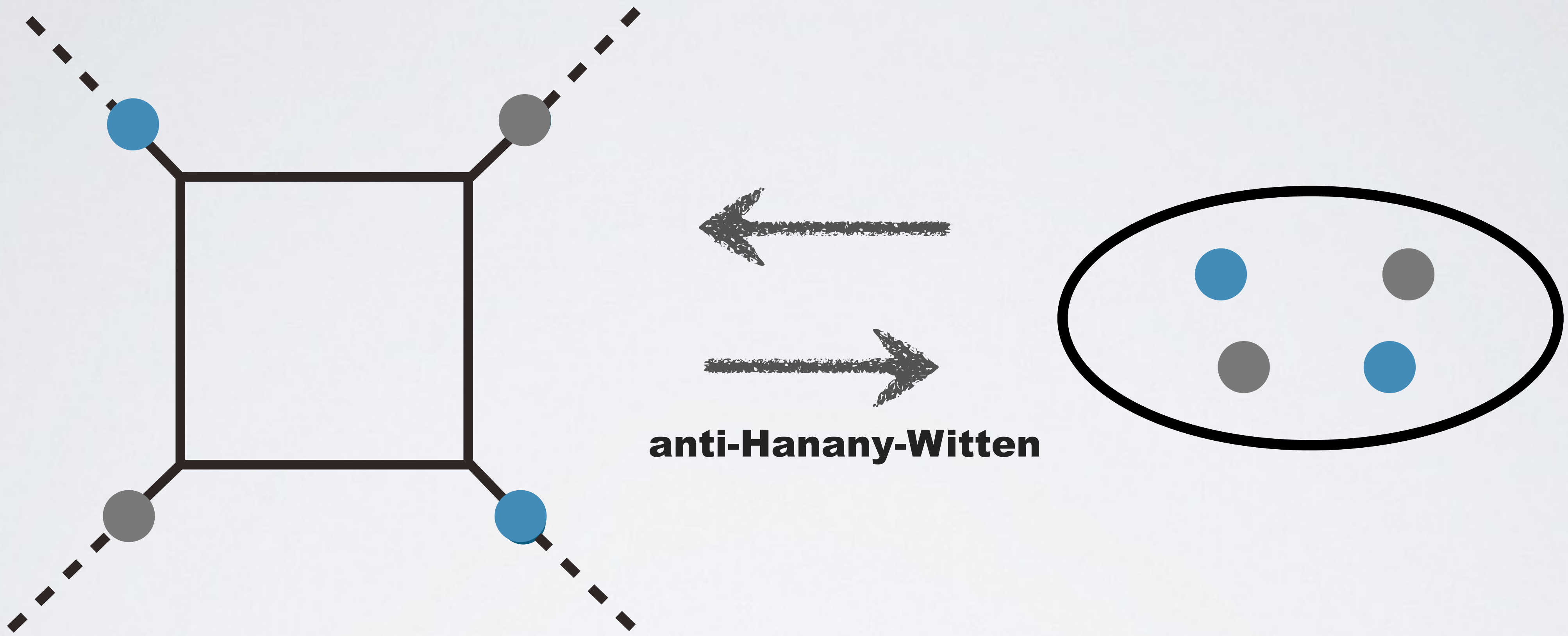


[p,q] line = geodesics of [p,q] 7-brane

5d N=1 SU(2) pure YM

[DeWolfe-Hanany-Iqbal-Katz, ’99]



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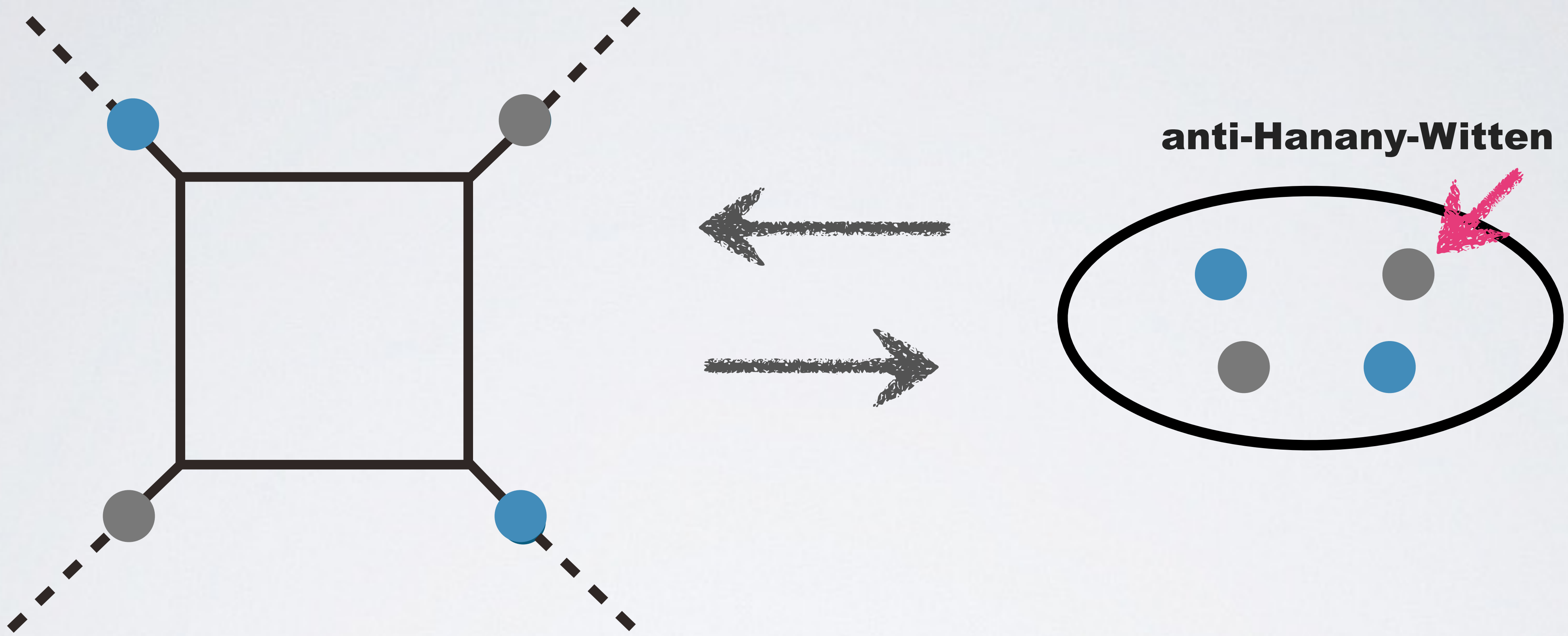

5d N=1 SU(2) pure YM

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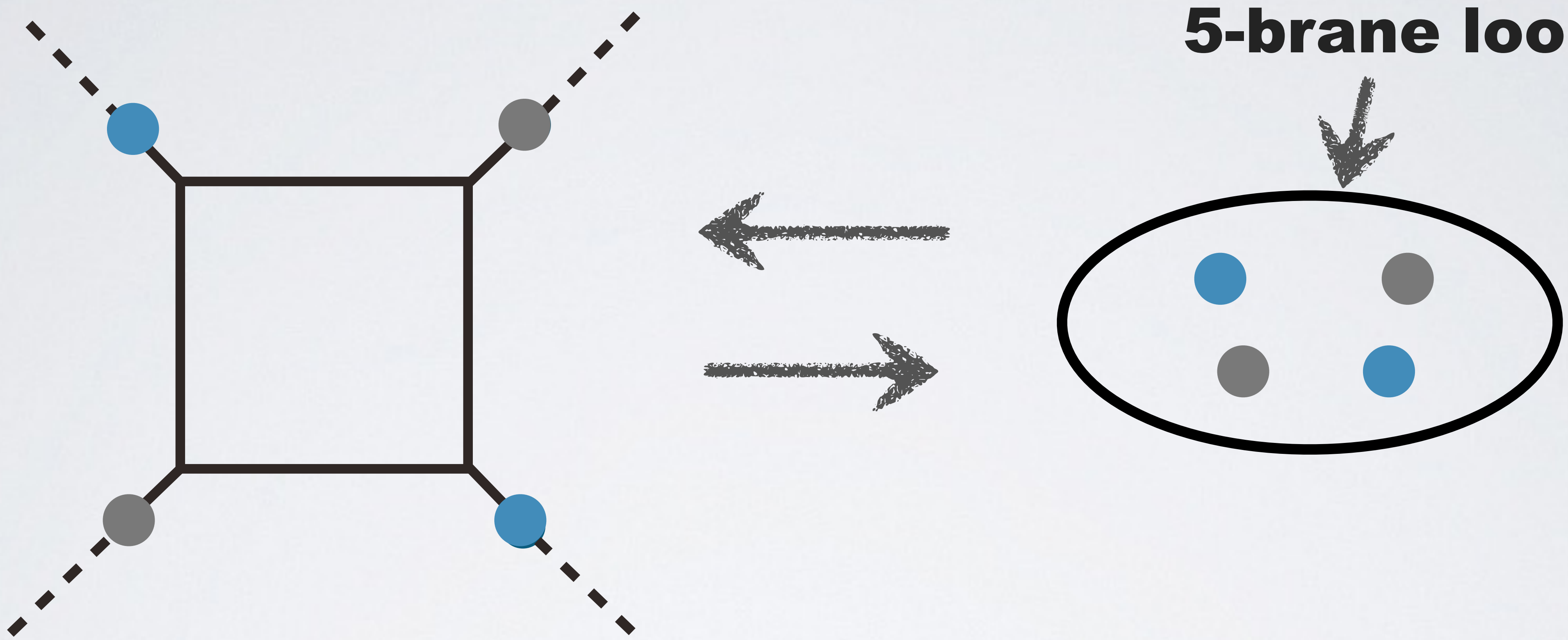

5d N=1 SU(2) pure YM

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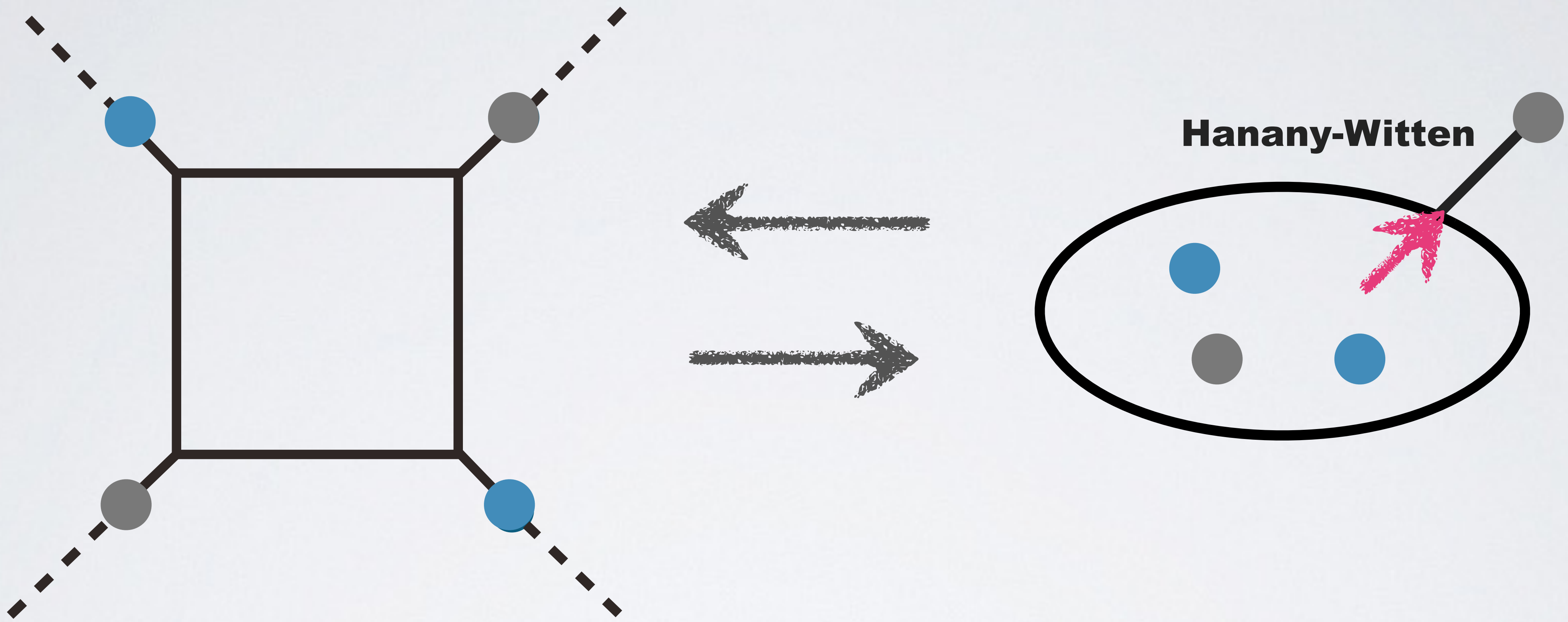
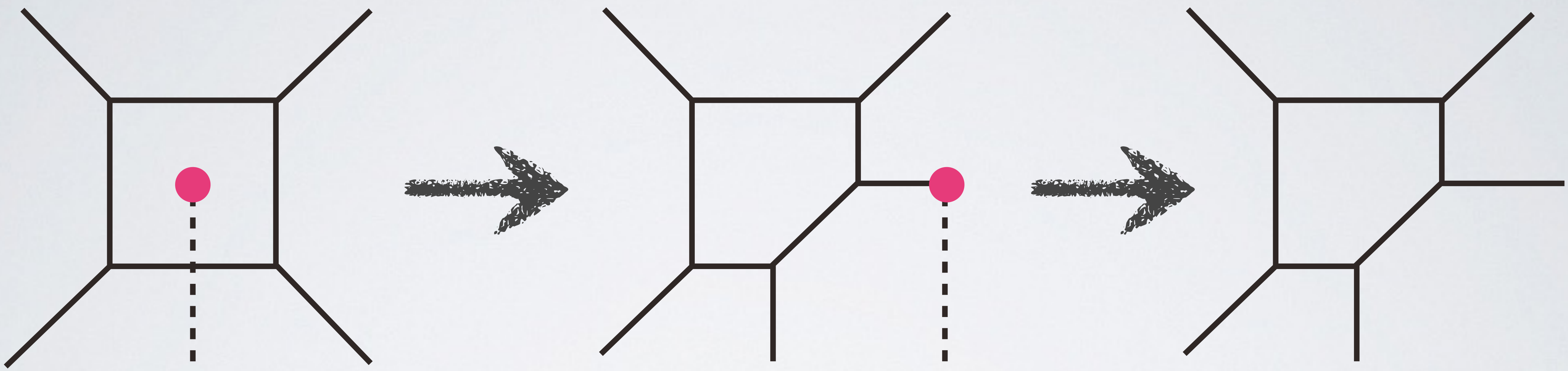


5d N=1 SU(2) SQCD

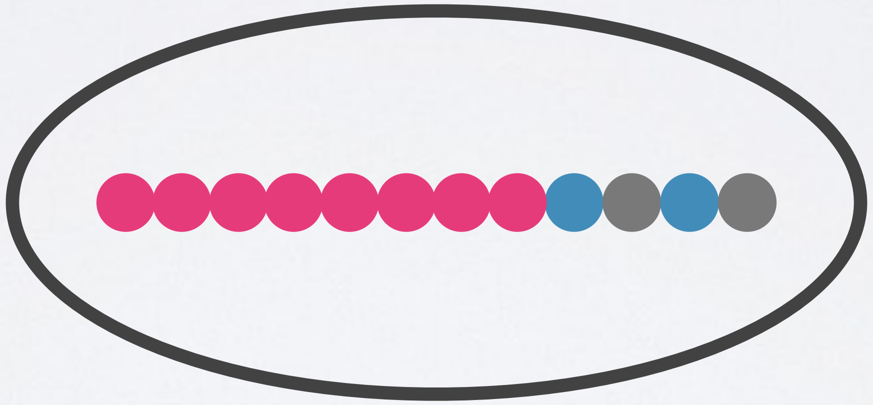
[DeWolfe-Hanany-Iqbal-Katz, '99]



planing a corner

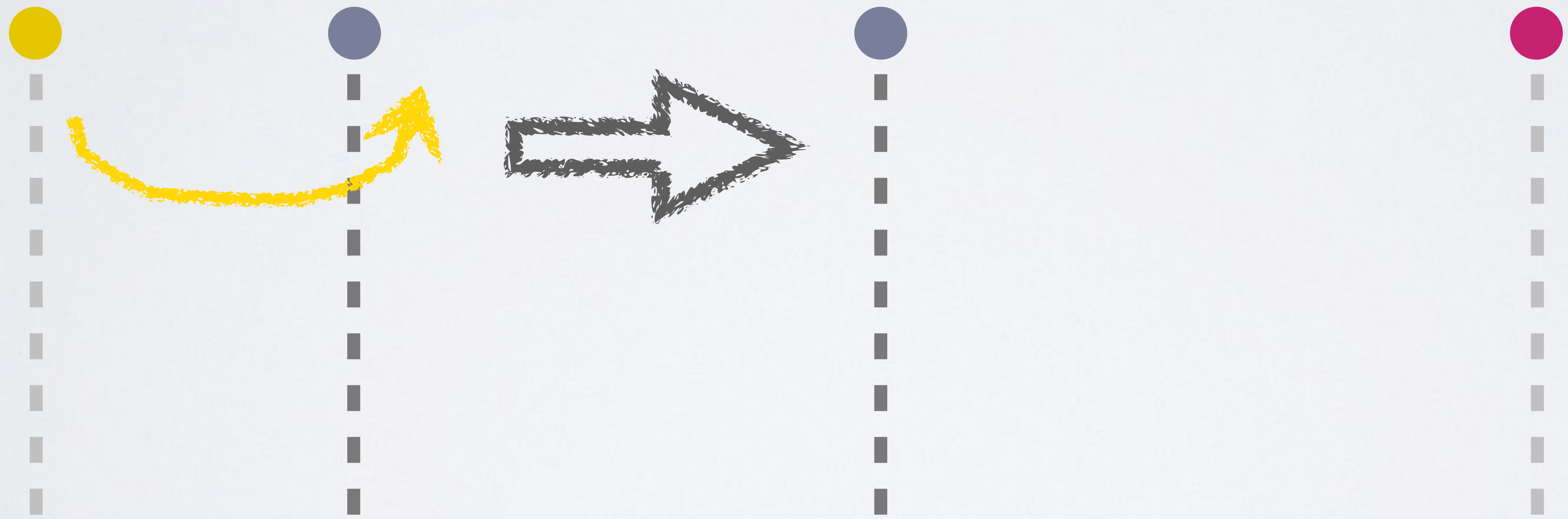
Generic configuration

$$(\mathbf{X}_{[1,0]})^{N_f}\ \mathbf{X}_{[1,-1]}\ \mathbf{X}_{[1,1]}\ \mathbf{X}_{[1,-1]}\ \mathbf{X}_{[1,1]}$$



Picard-Lefschetz transformation

$$\begin{pmatrix} p \\ q \end{pmatrix} \quad \begin{pmatrix} p' \\ q' \end{pmatrix} \qquad \begin{pmatrix} p' \\ q' \end{pmatrix} \quad \begin{pmatrix} p \\ q \end{pmatrix} + \det \begin{pmatrix} p & p' \\ q & q' \end{pmatrix} \begin{pmatrix} p' \\ q' \end{pmatrix}$$



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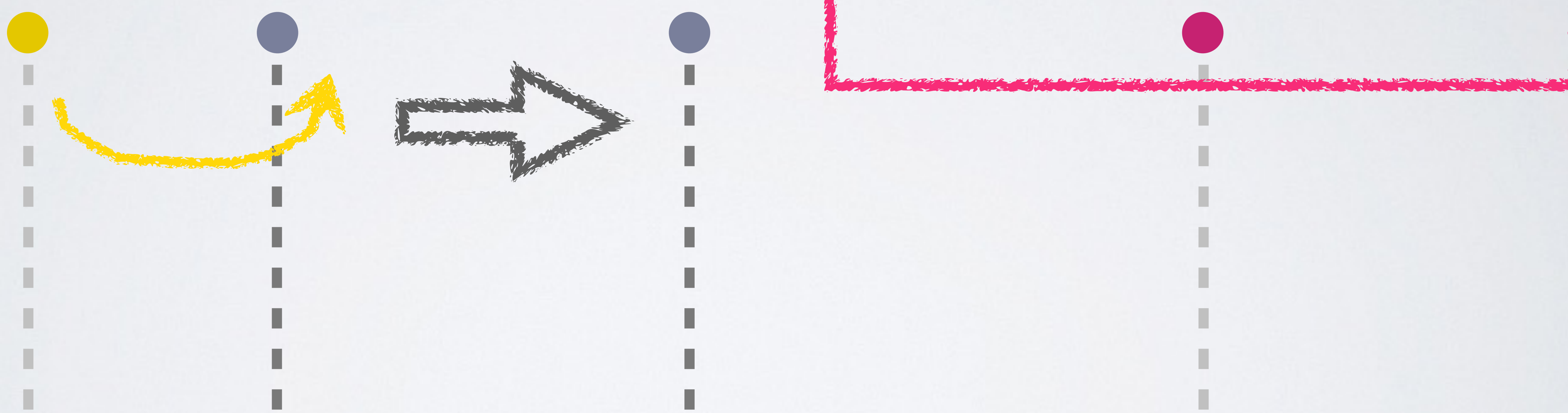
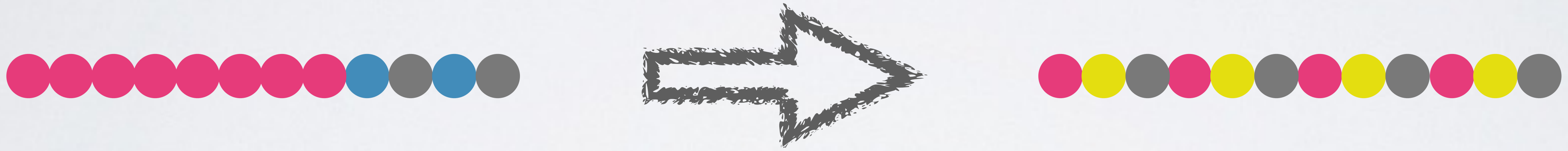
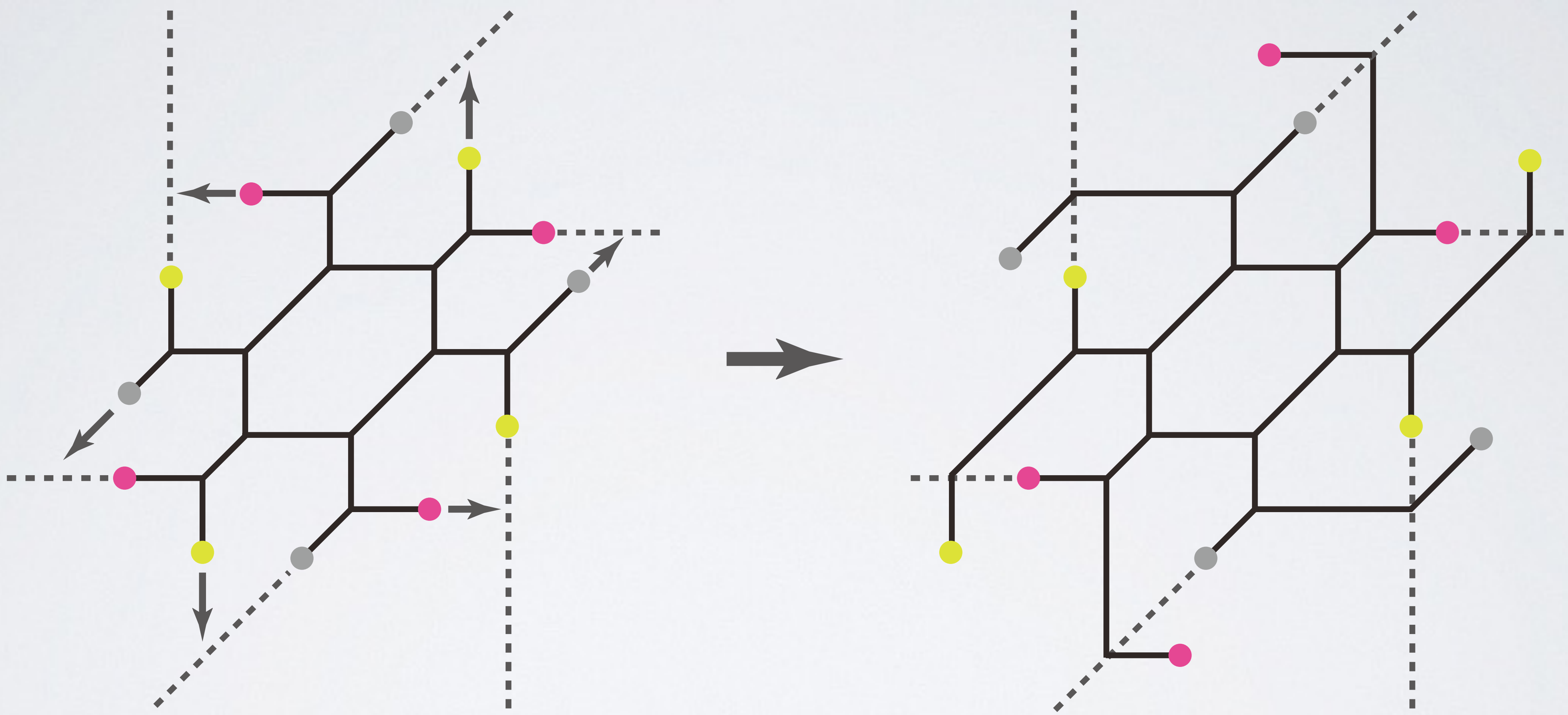


Tao-nization

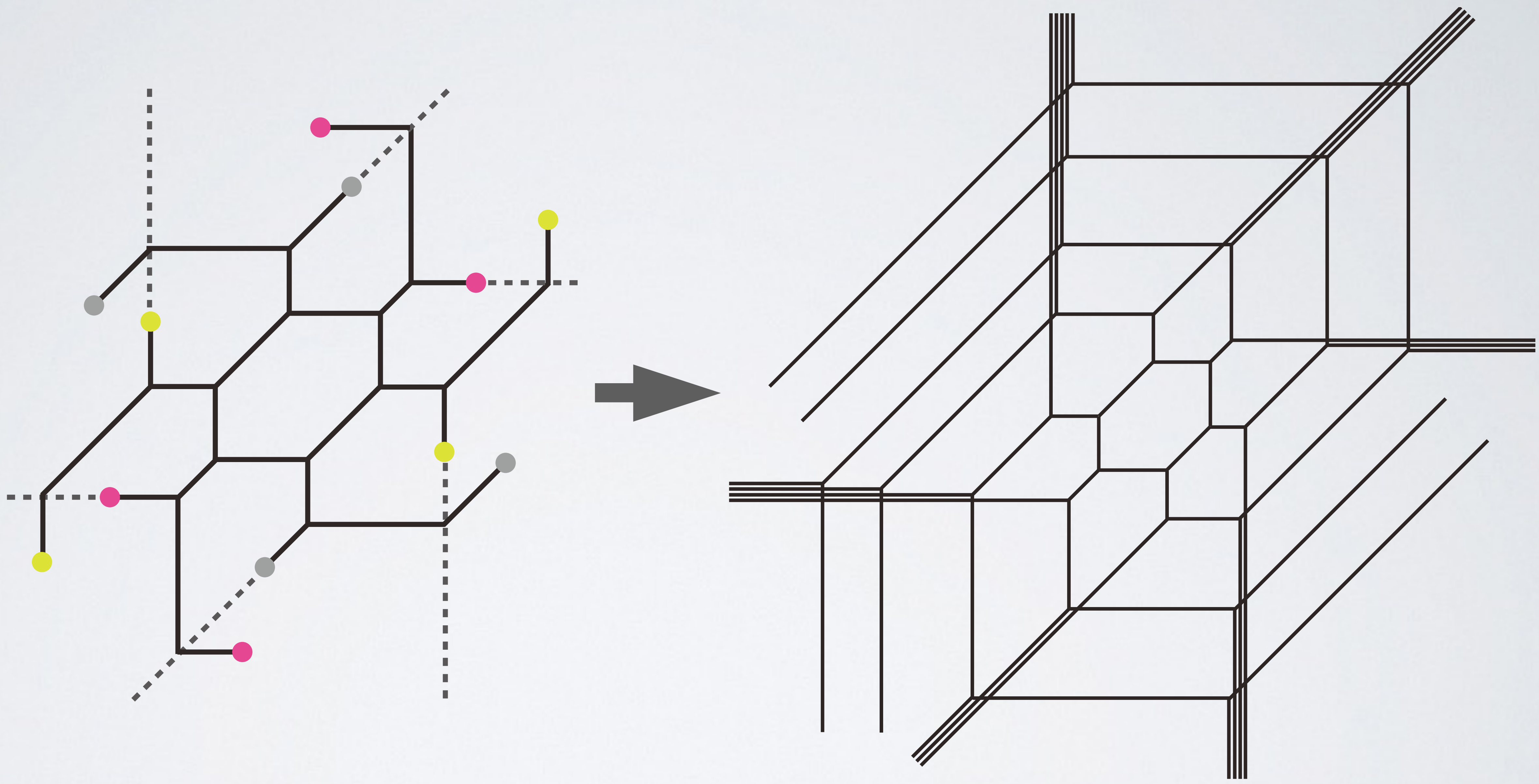
(1,0) (0,1) (1,1) configuration



Tao-nization

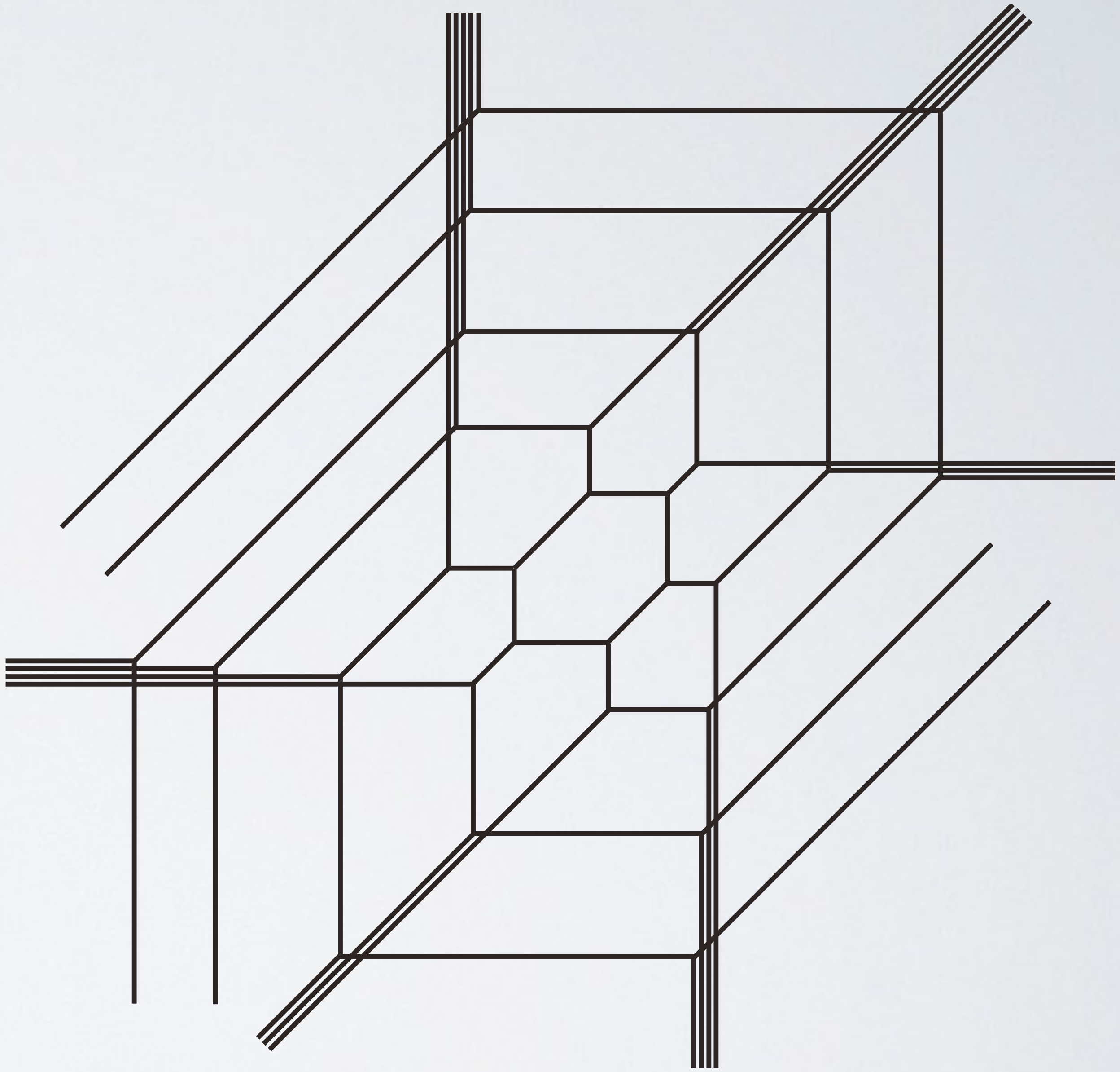


Tao-nization



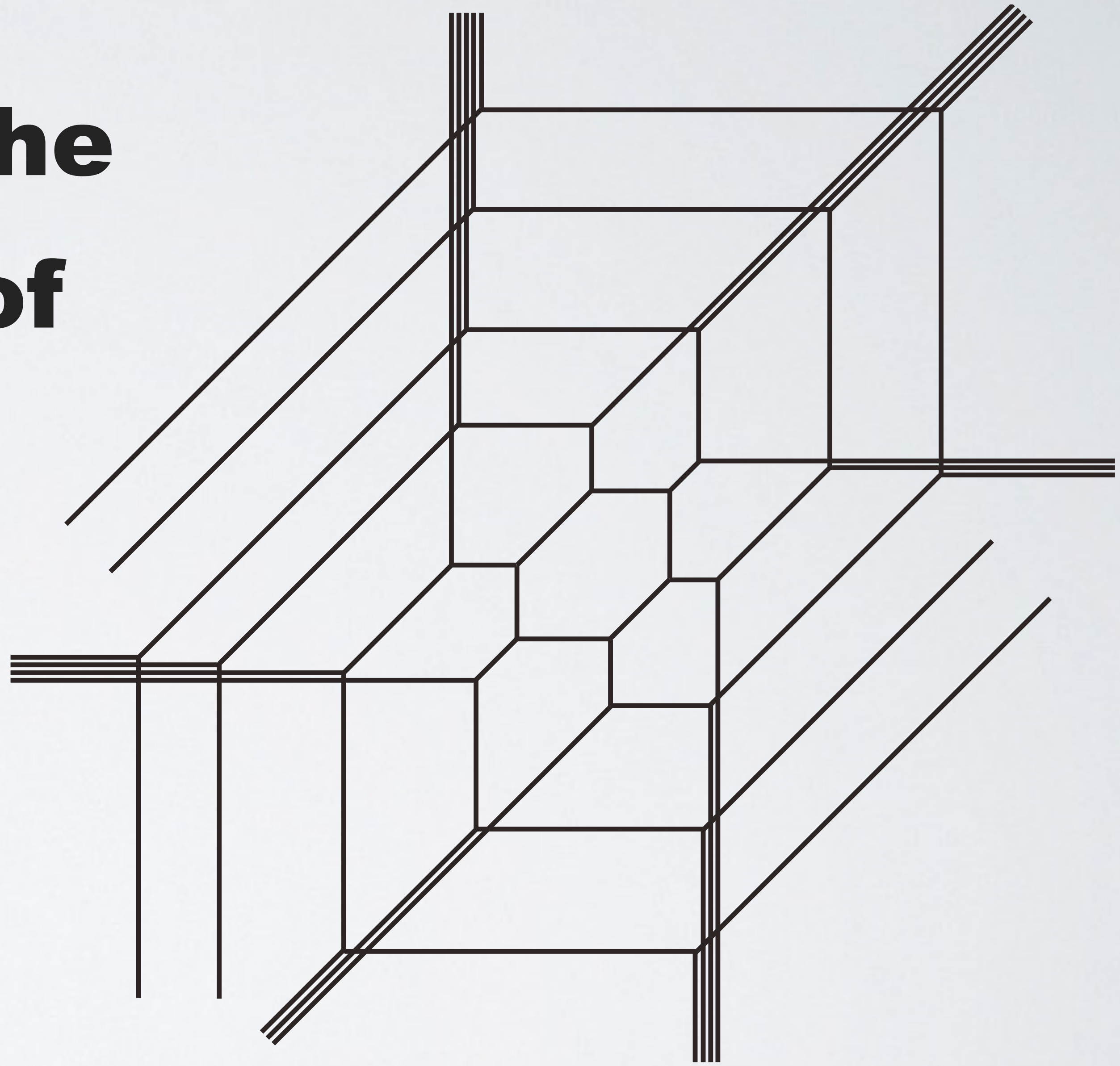
IIB realization of 6d E-string Theory

Tao brane web



IIB realization of 6d E-string Theory

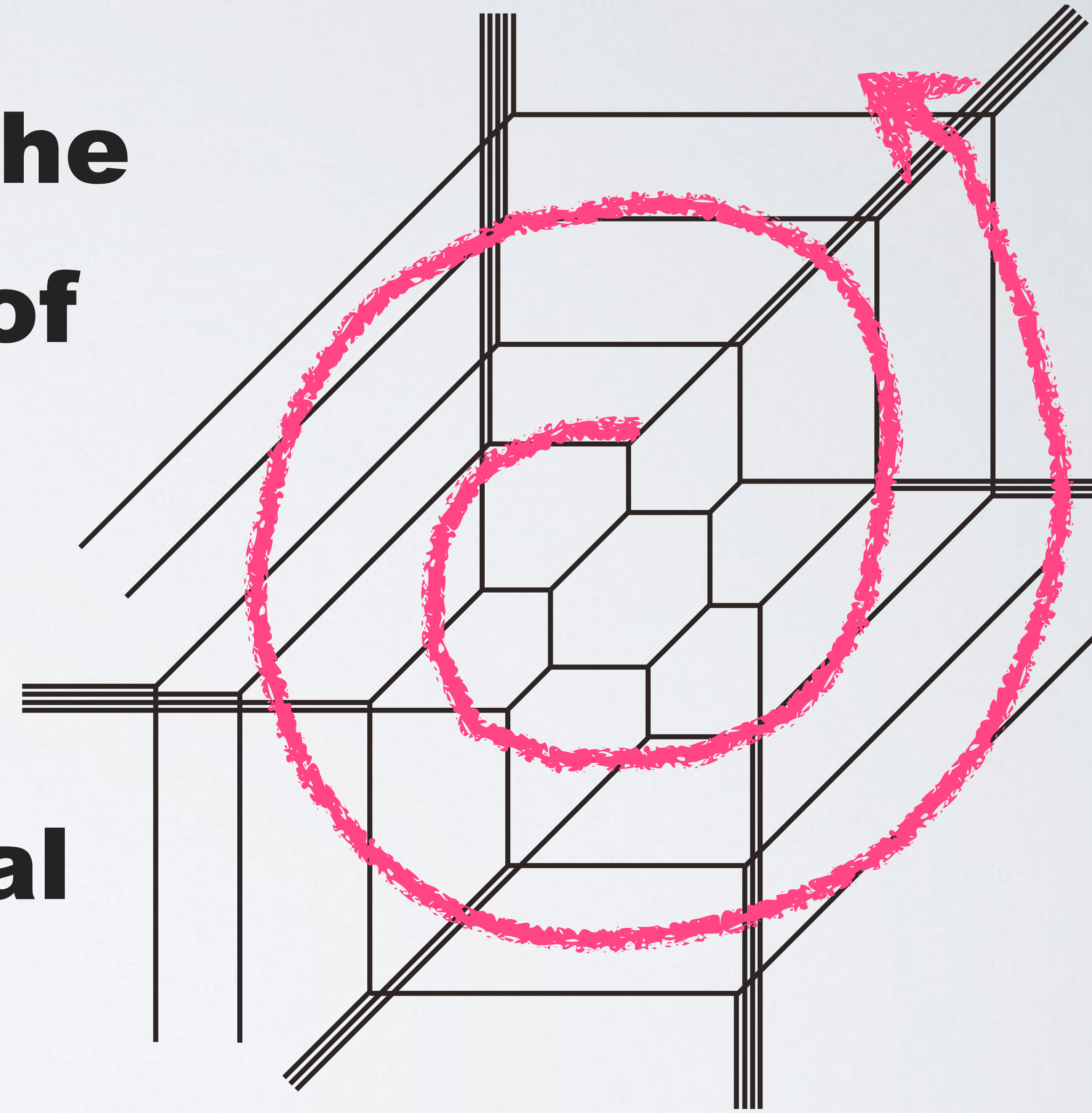
6d E-string theory is the world volume theory of the 5-brane web



IIB realization of 6d E-string Theory

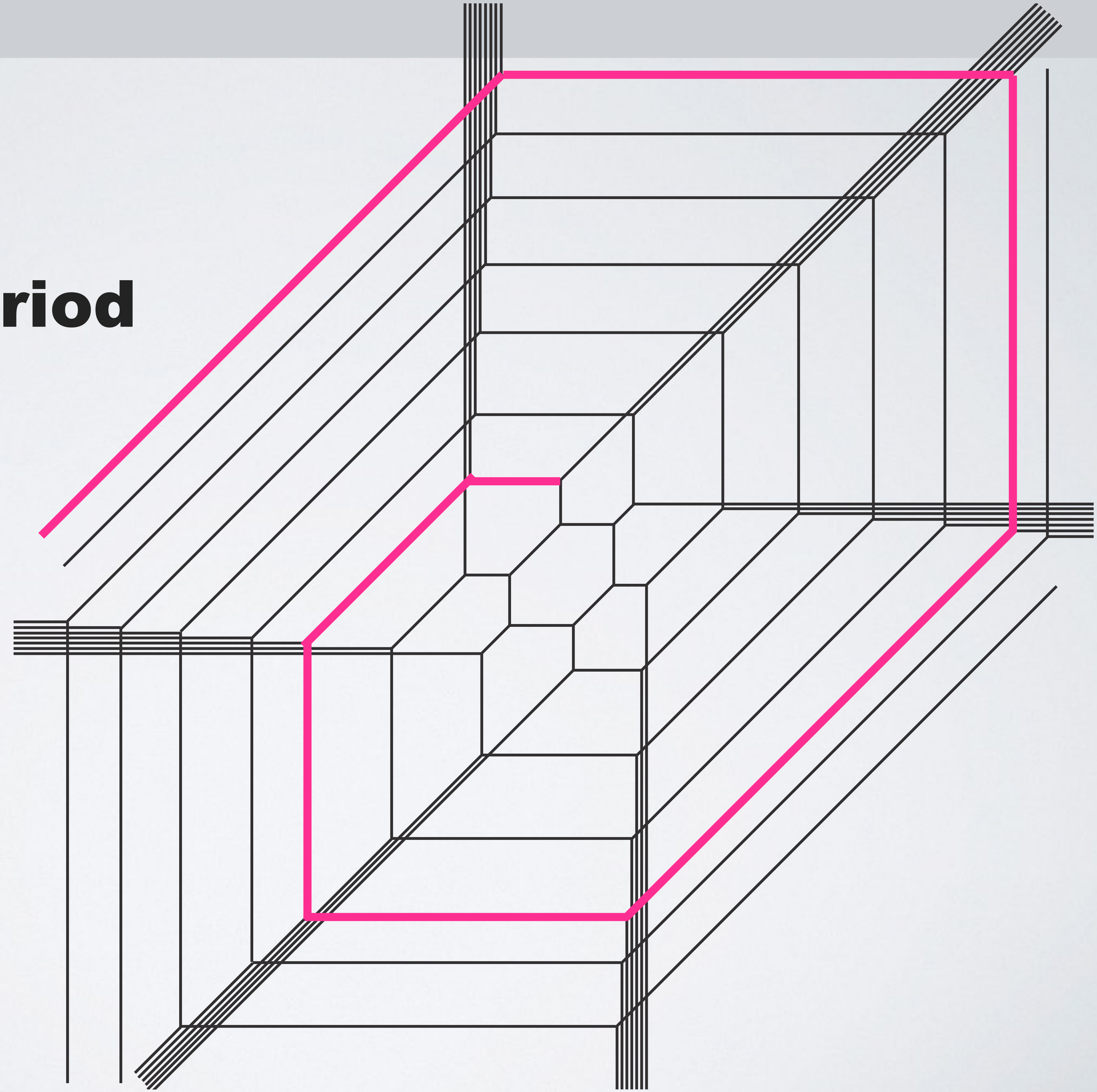
6d E-string theory is the world volume theory of the 5-brane web

6-th direction is generated by the spiral direction (KK=winding)



Tao Web

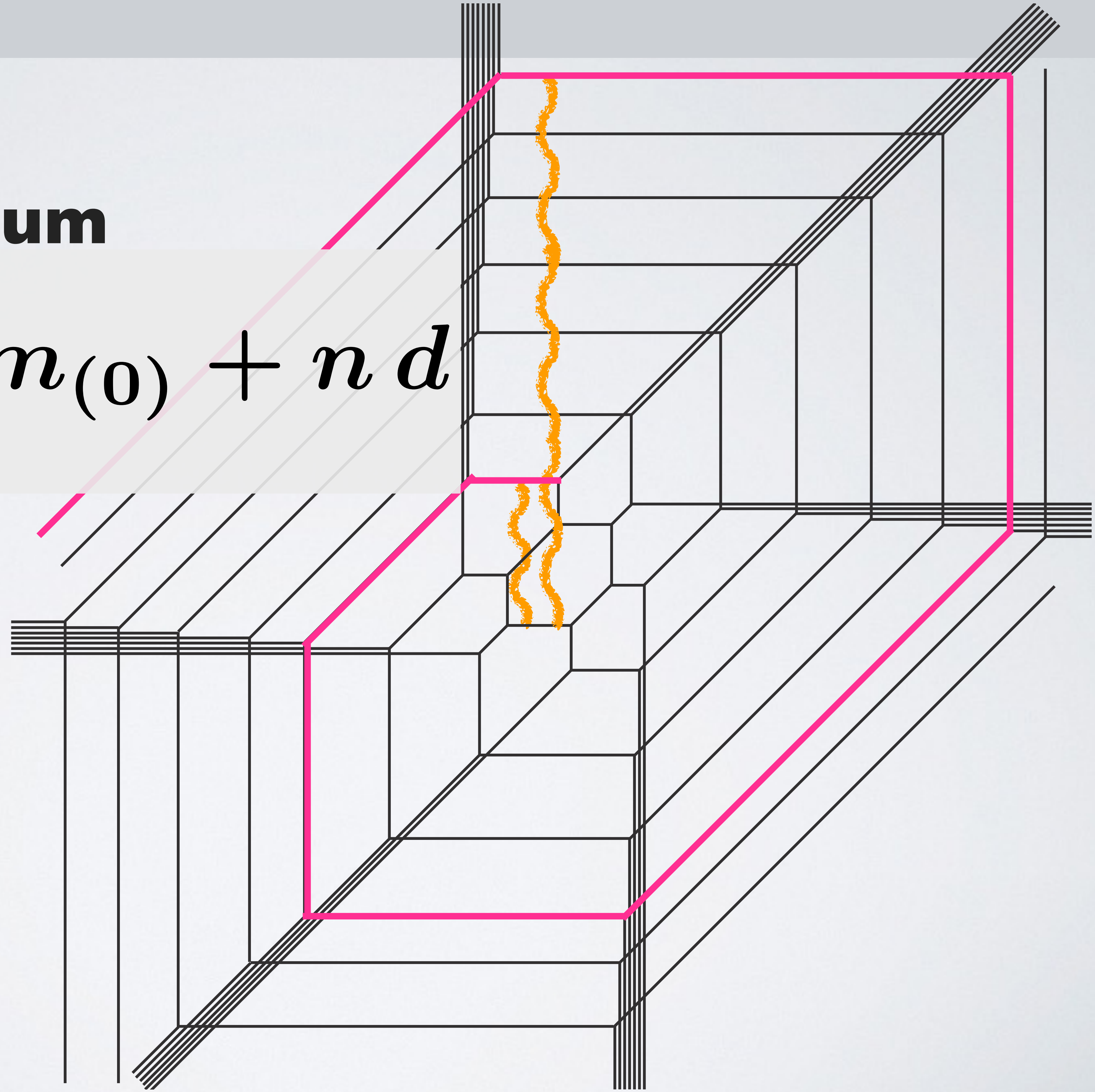
spiral with constant period



Tao Web

BPS spectrum

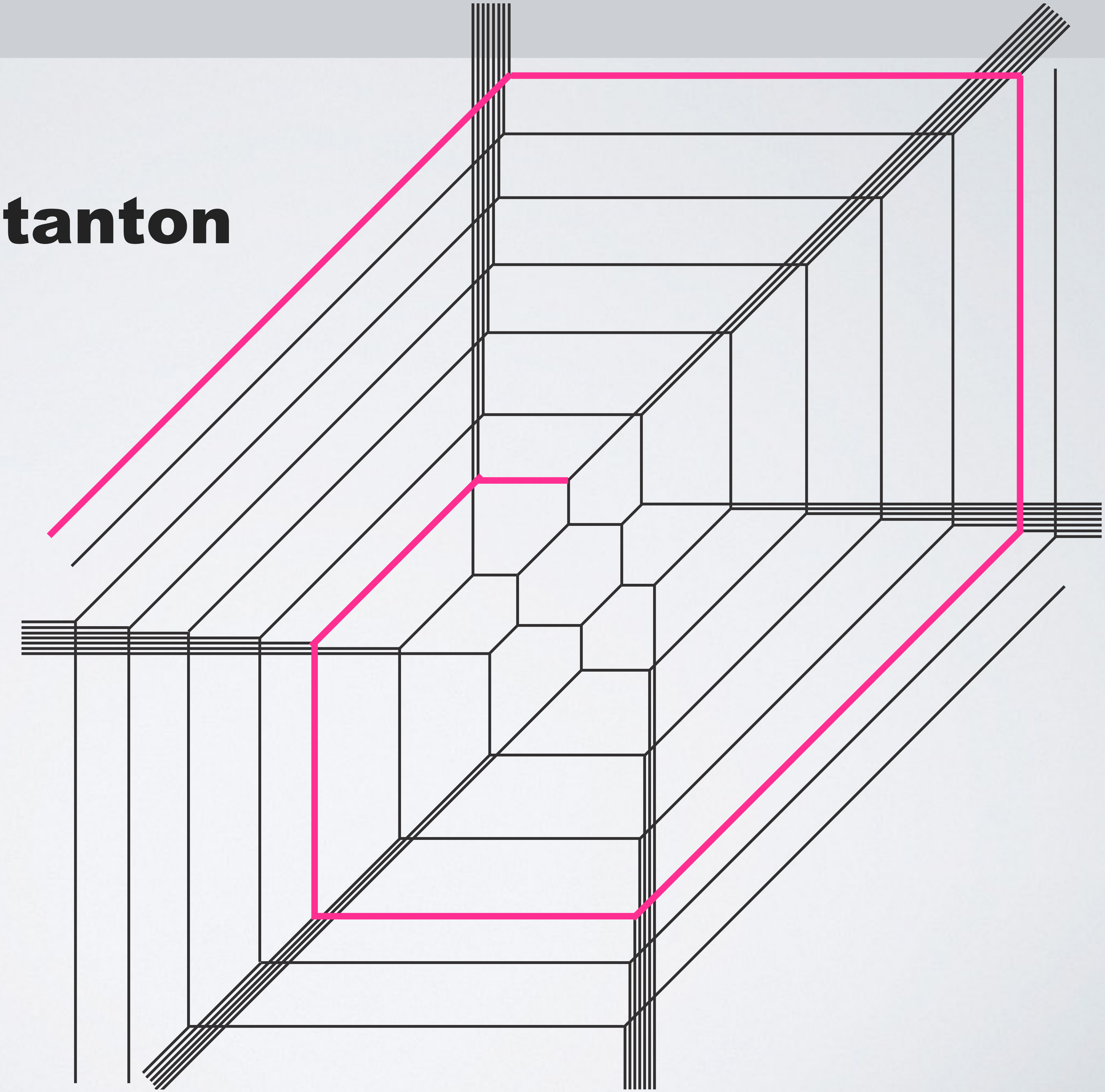
$$m_{(n)} = m_{(0)} + n\,d$$



Tao Web

radius = instanton

$$d = \frac{1}{g^2}$$



6d nature of a Tao Web

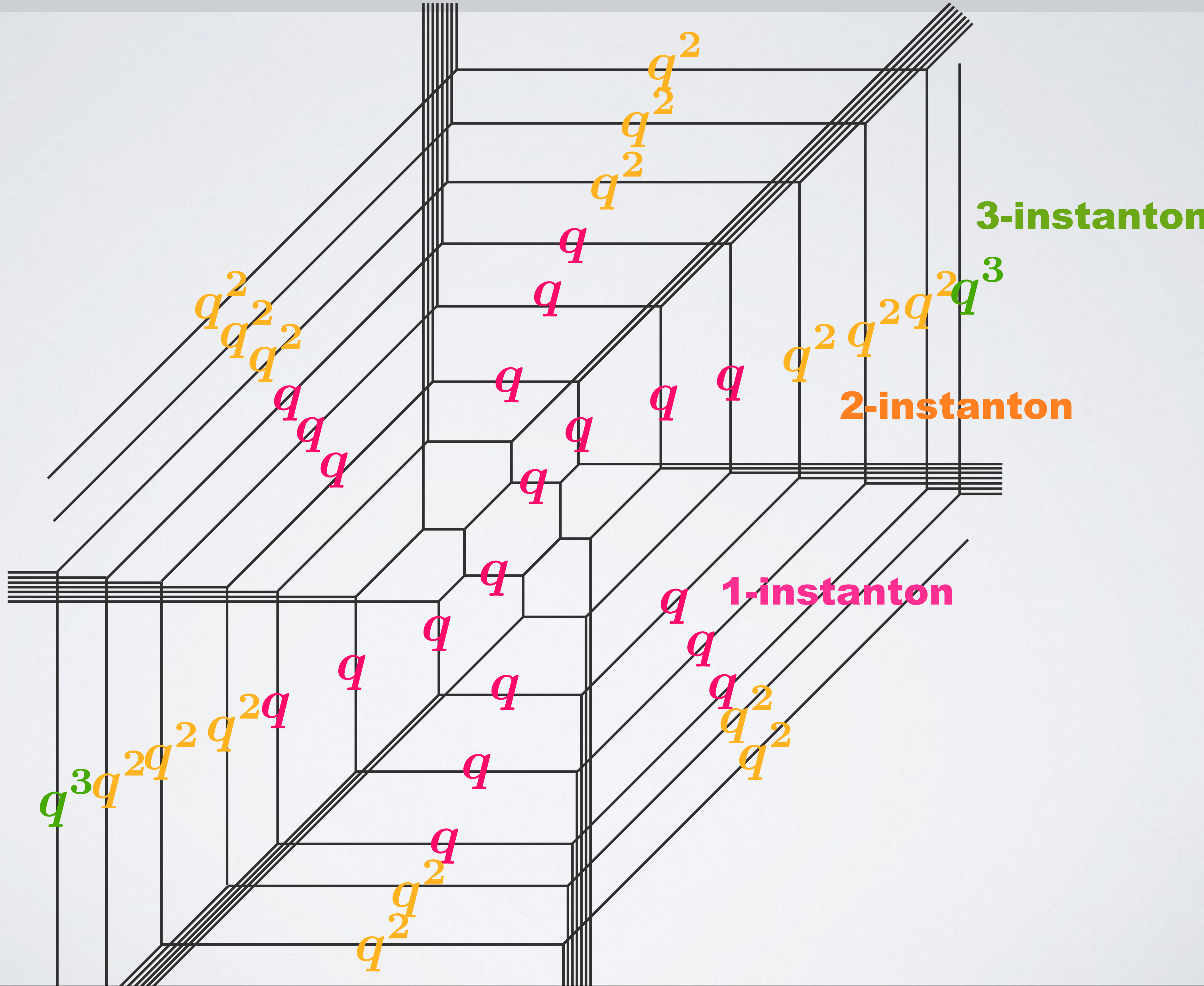
(T-dual) radius = instanton

$$R \propto \frac{1}{d} = g^2$$

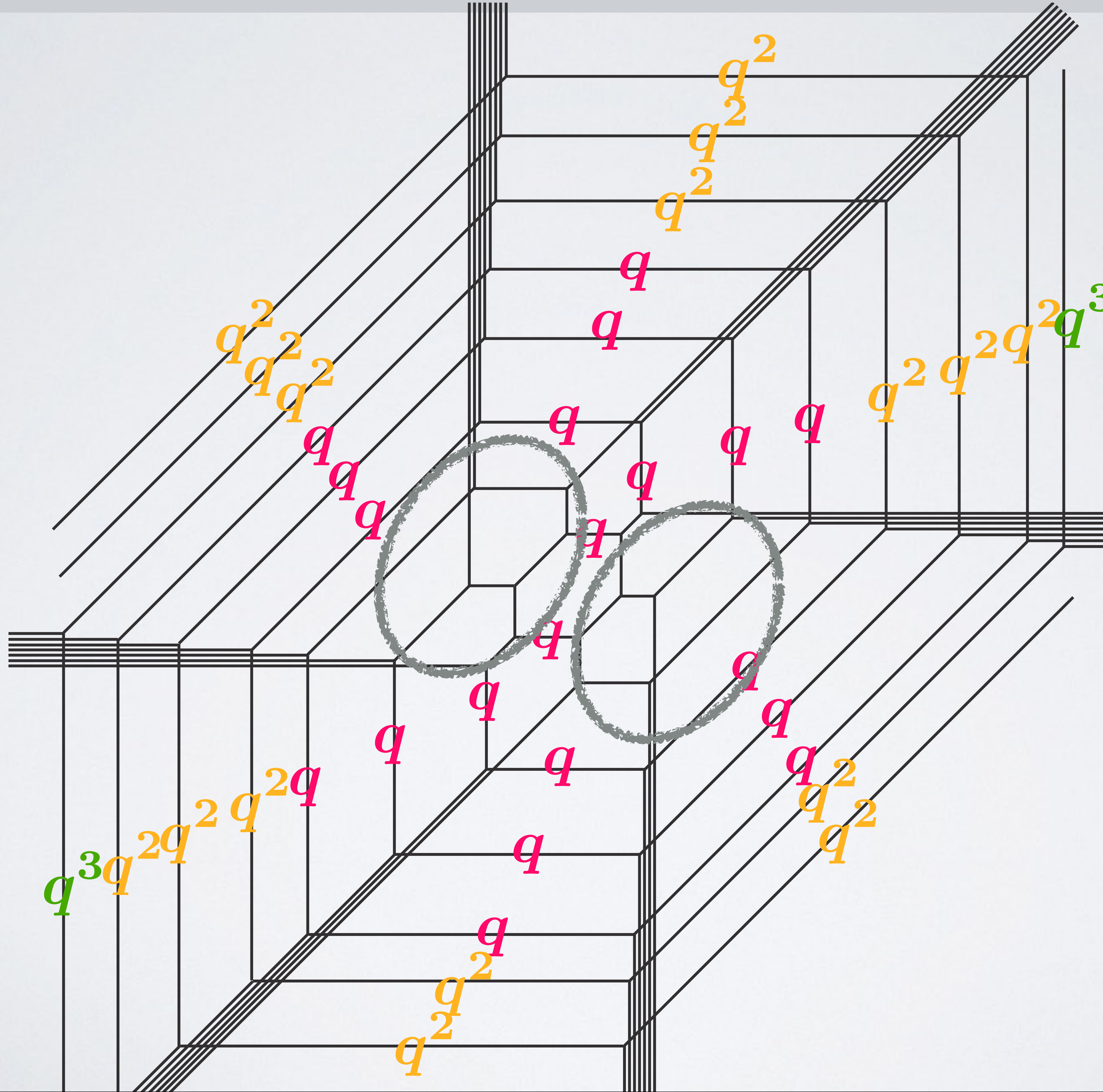
↓

KK (winding) modes = instanton solitons

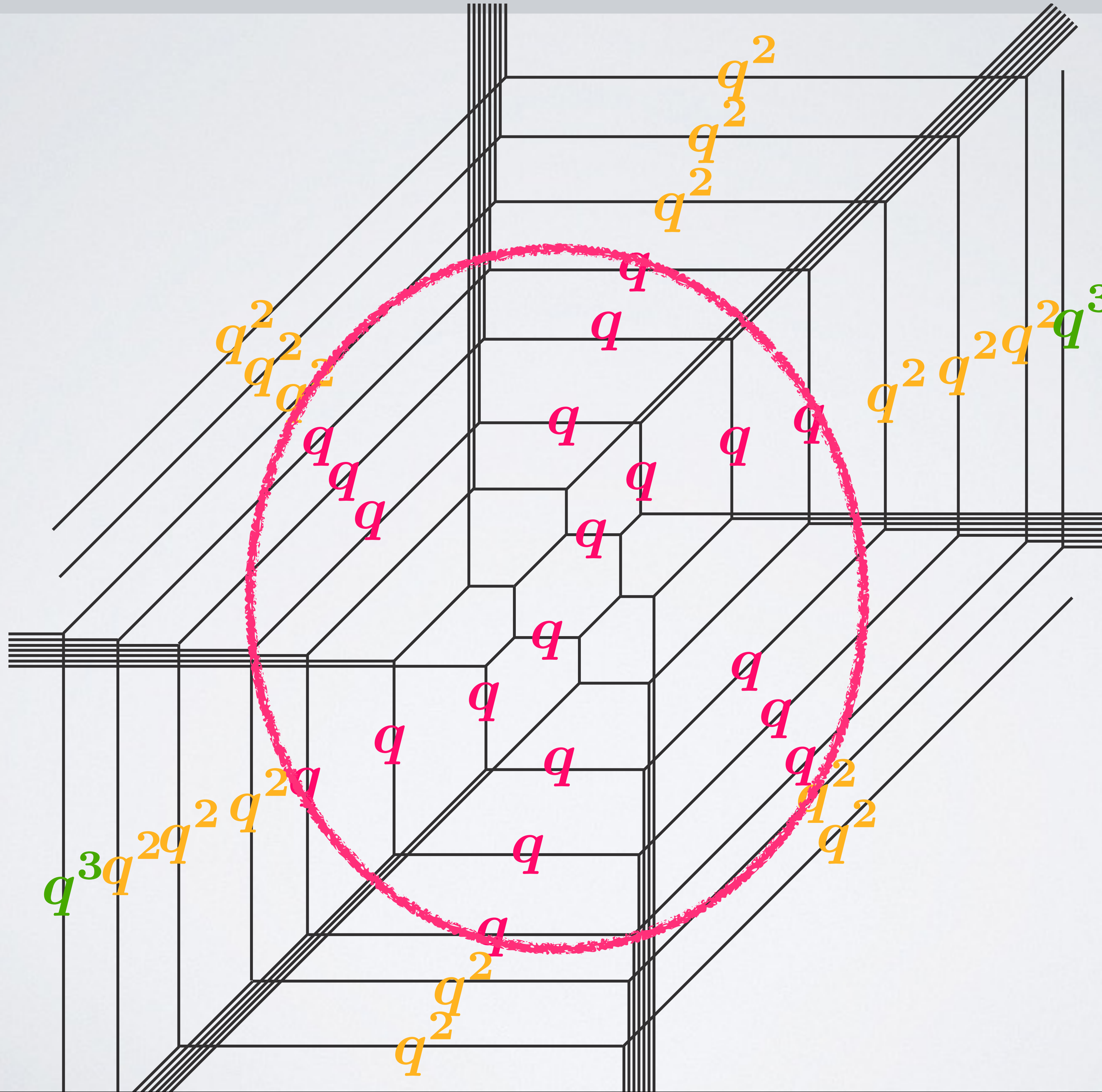
Instanton dependence of Tao web



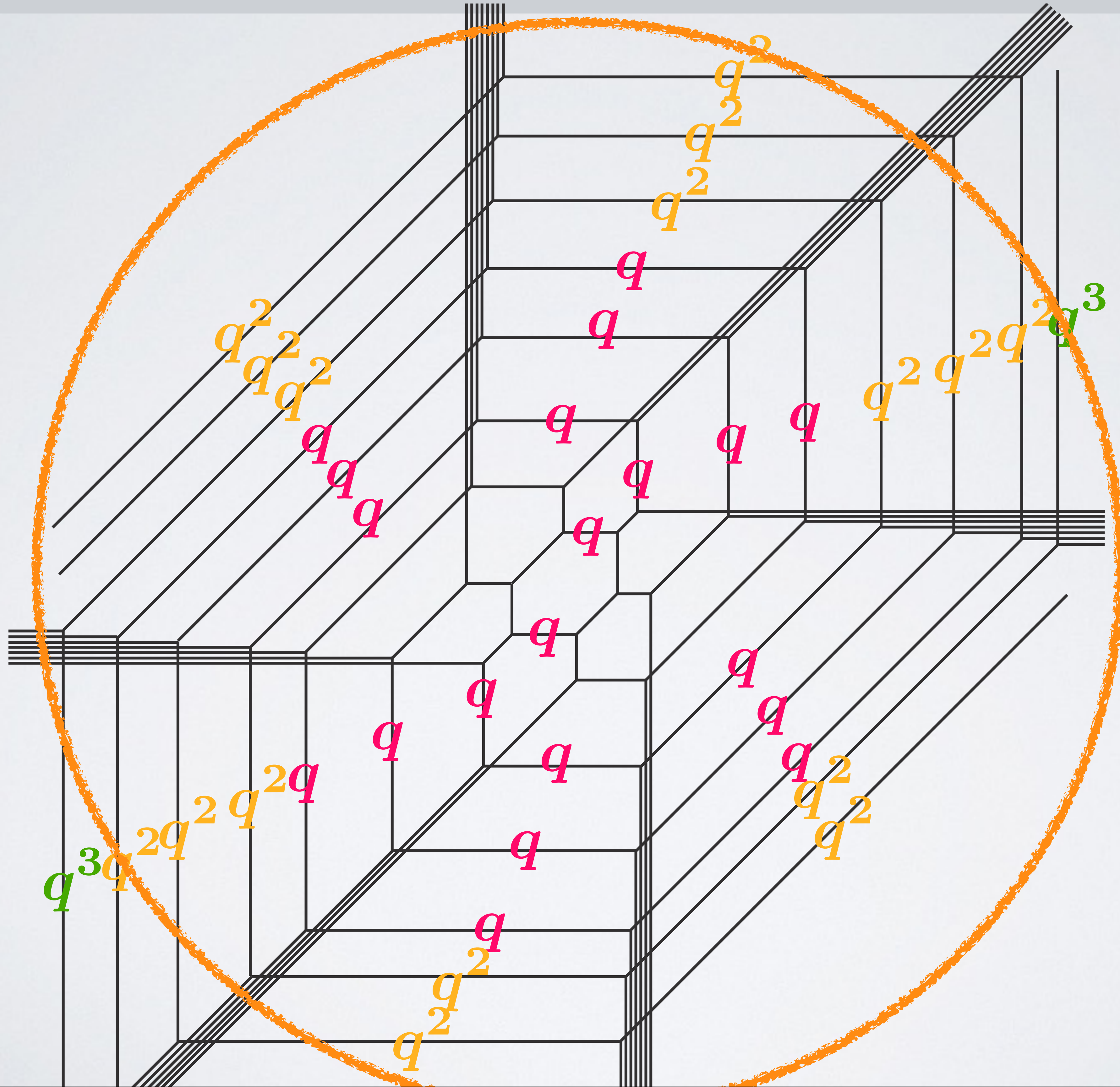
perturbative contribution



1-instanton contribution



2-instanton contribution



Application to Anomaly Polynomial

Anomaly polynomial

8 form describing 6d anomalies

cf [Ohmori-Shimizu-Tachikawa-Yonekura, '14]

Anomaly polynomial

8 form describing 6d anomalies

cf [Ohmori-Shimizu-Tachikawa-Yonekura, '14]

superconformal index leads to anomaly polynomial (conjecture?) [Bobev-Bullimore-Kim, '15]

Anomaly polynomial

8 form describing 6d anomalies

cf [Ohmori-Shimizu-Tachikawa-Yonekura, '14]

superconformal index leads to anomaly polynomial (conjecture?) [Bobev-Bullimore-Kim, '15]

partition function on S^5 can be computed by topological strings (conjecture) [Lockhart-Vafa, '12]

$$Z_{S^5} = \int da\, Z \cdot Z \cdot Z$$

Anomaly polynomial

8 form describing 6d anomalies

cf [Ohmori-Shimizu-Tachikawa-Yonekura, '14]

superconformal index leads to anomaly polynomial (conjecture?) [Bobev-Bullimore-Kim, '15]

partition function on S^5 can be computed by topological strings (conjecture) [Lockhart-Vafa, '12]

Applying Tao by assuming these things

What we can learn

Test of Tao web description of 6d SCFT

Studying BBK conjecture

New perspective for topological strings

Anomaly polynomial

superconformal index leads to anomaly polynomial

[Bobev-Bullimore-Kim, '15]

$$\lim_{R\to\infty} -\log Z^{S^1(R)\times S^5} = \int A_8$$

Anomaly polynomial

superconformal index leads to anomaly polynomial

[Bobev-Bullimore-Kim, '15]

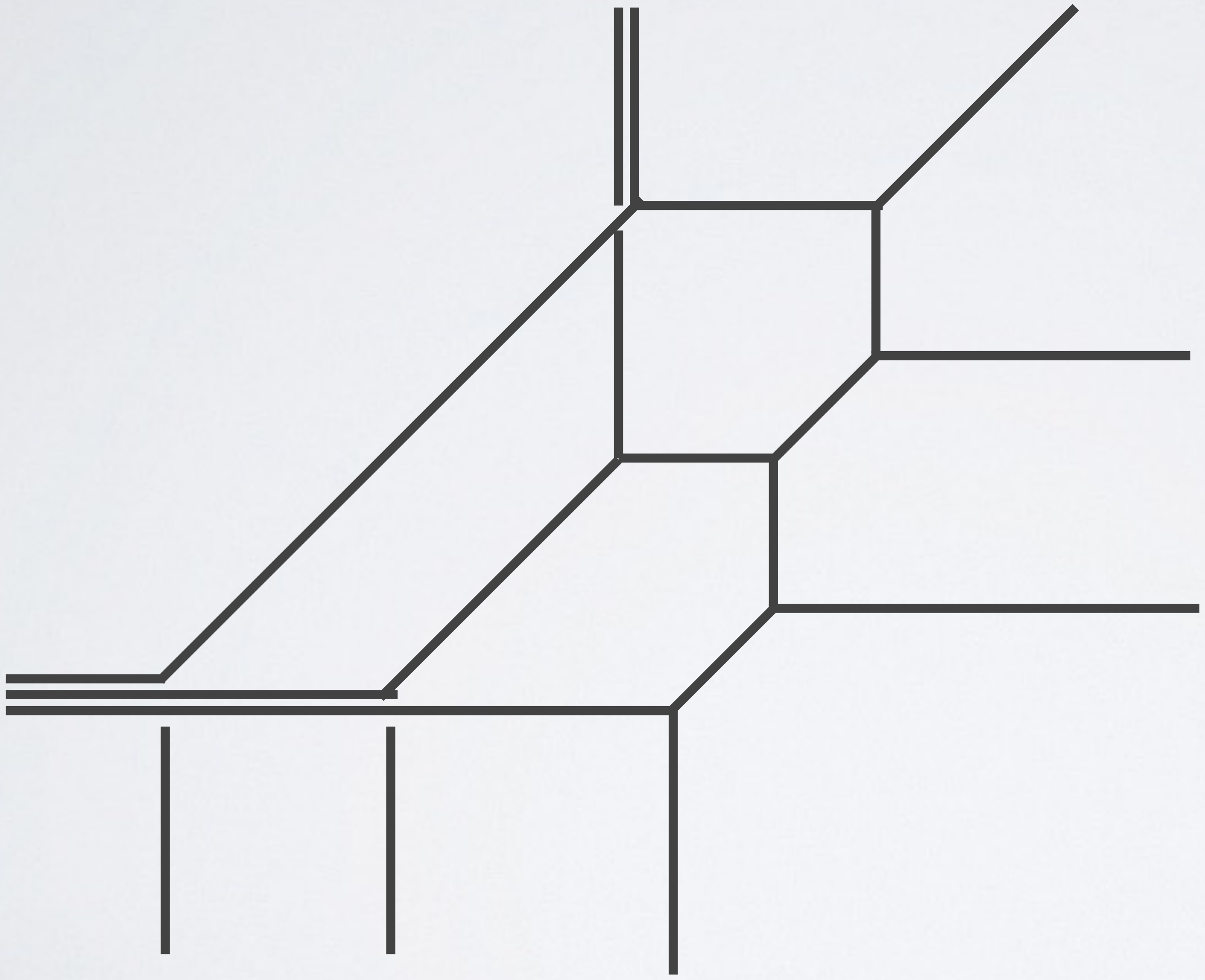
$$\lim_{R\to\infty} -\log Z^{S^1(R)\times S^5} = \int A_8$$

superconformal index comes from Tao web

$$Z^{S^1(R)\times S^5} = Z^{S^5}_{R=g^2}$$

E-string case

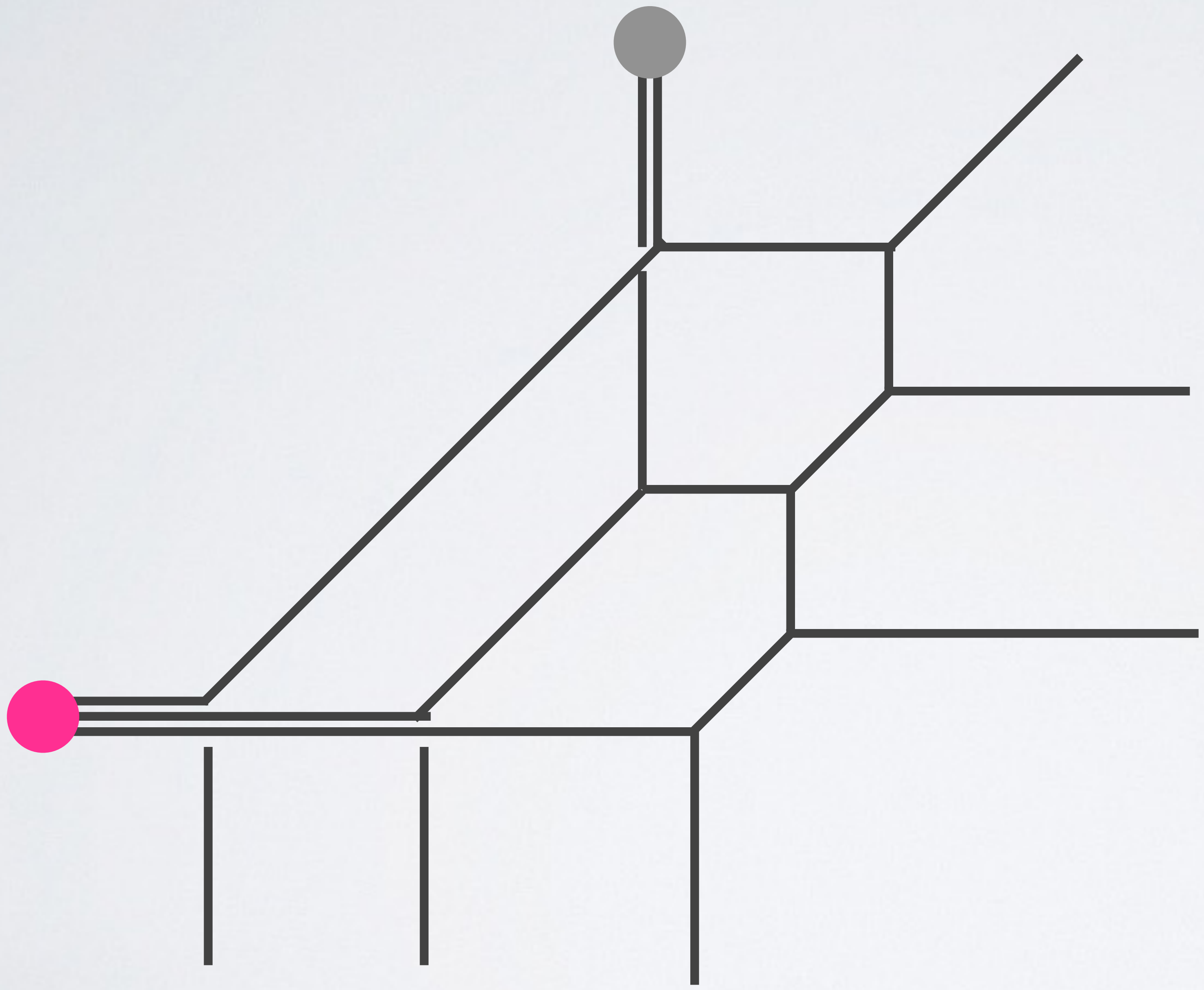
Following [Bobev-Bullimore-Kim, '15] we focus only on perturbative part



(half of) sub-diagram that gives perturbative index

E-string case

TRICK: deformation by 7-branes



E-string case

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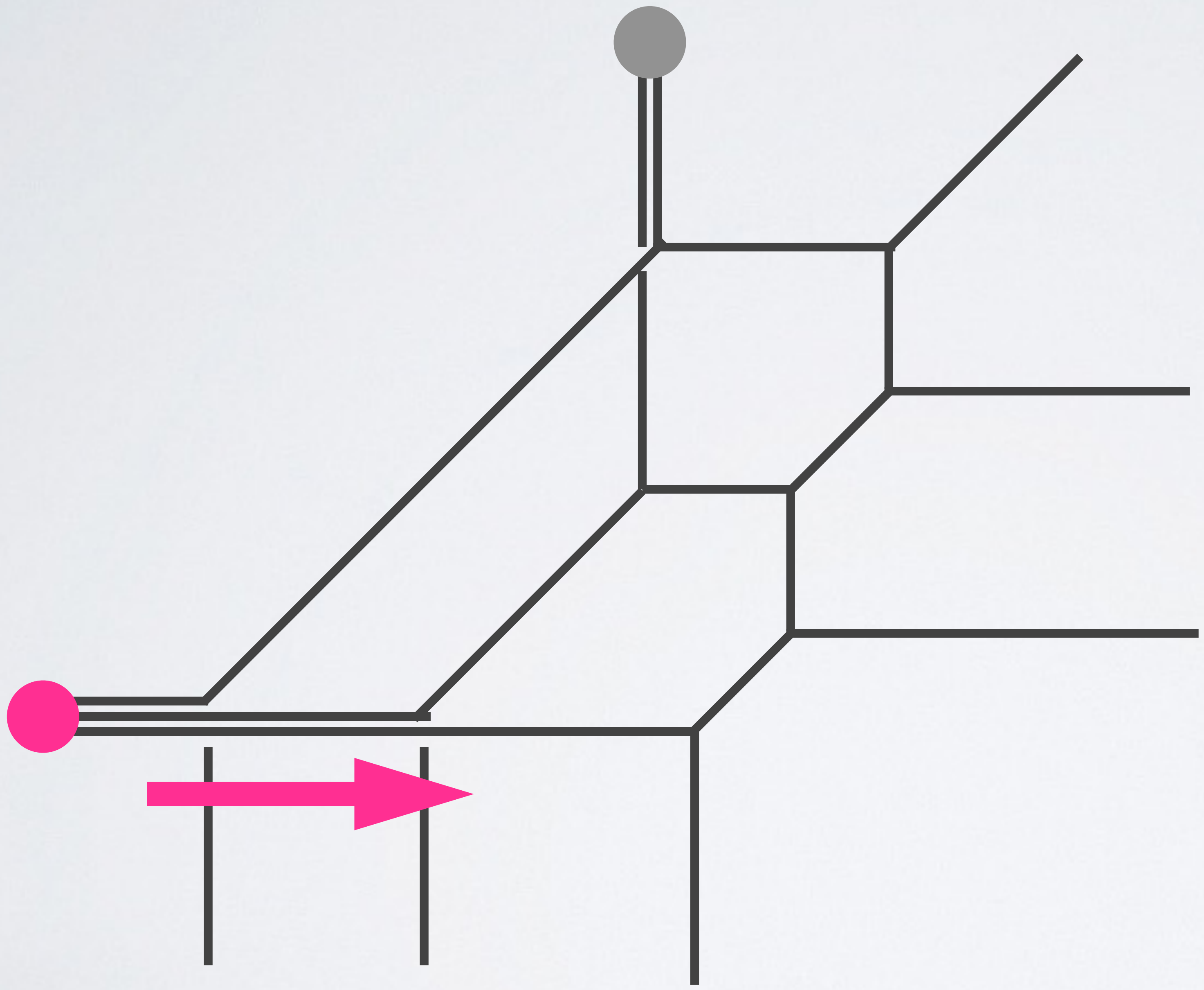

E-string case

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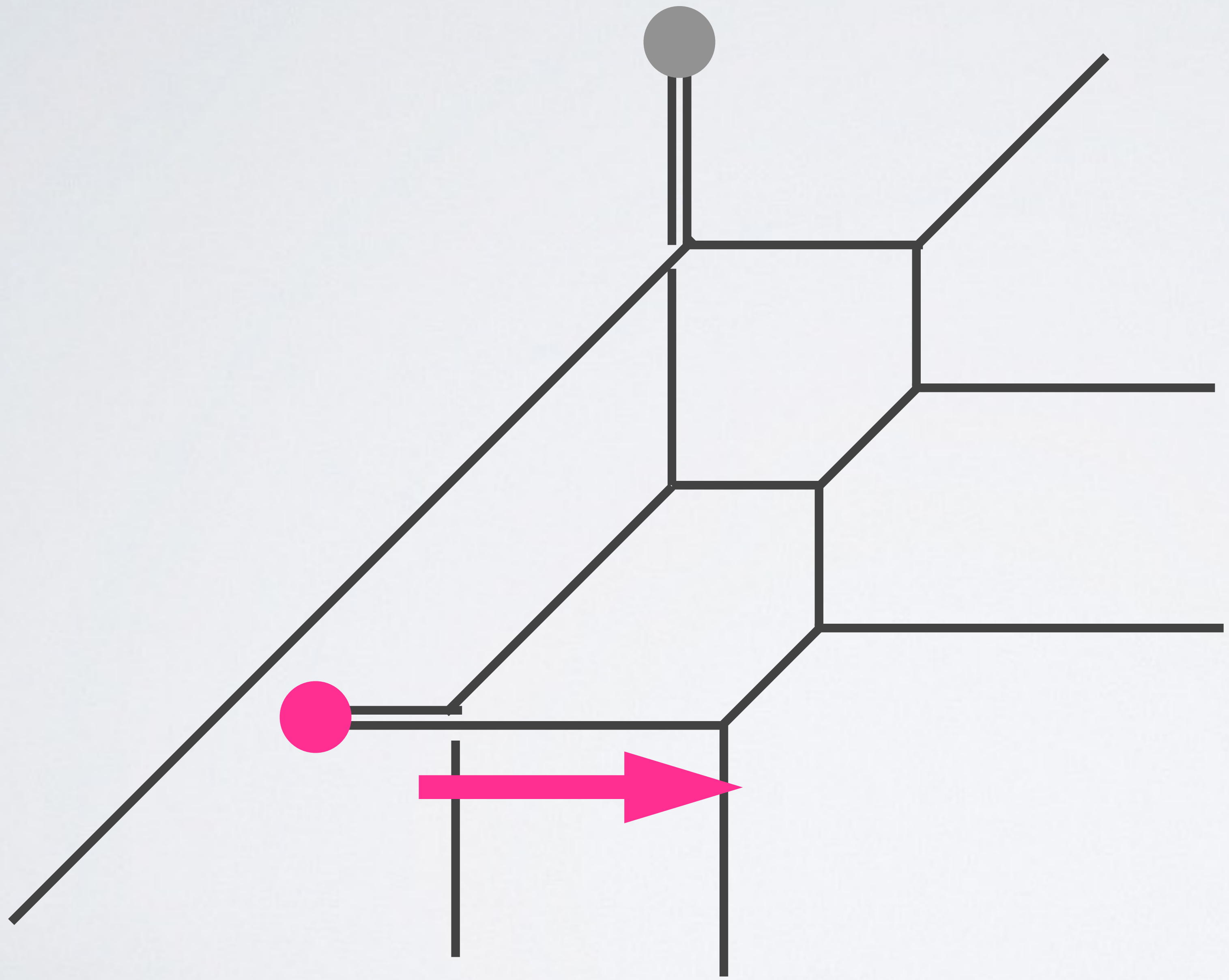

E-string case

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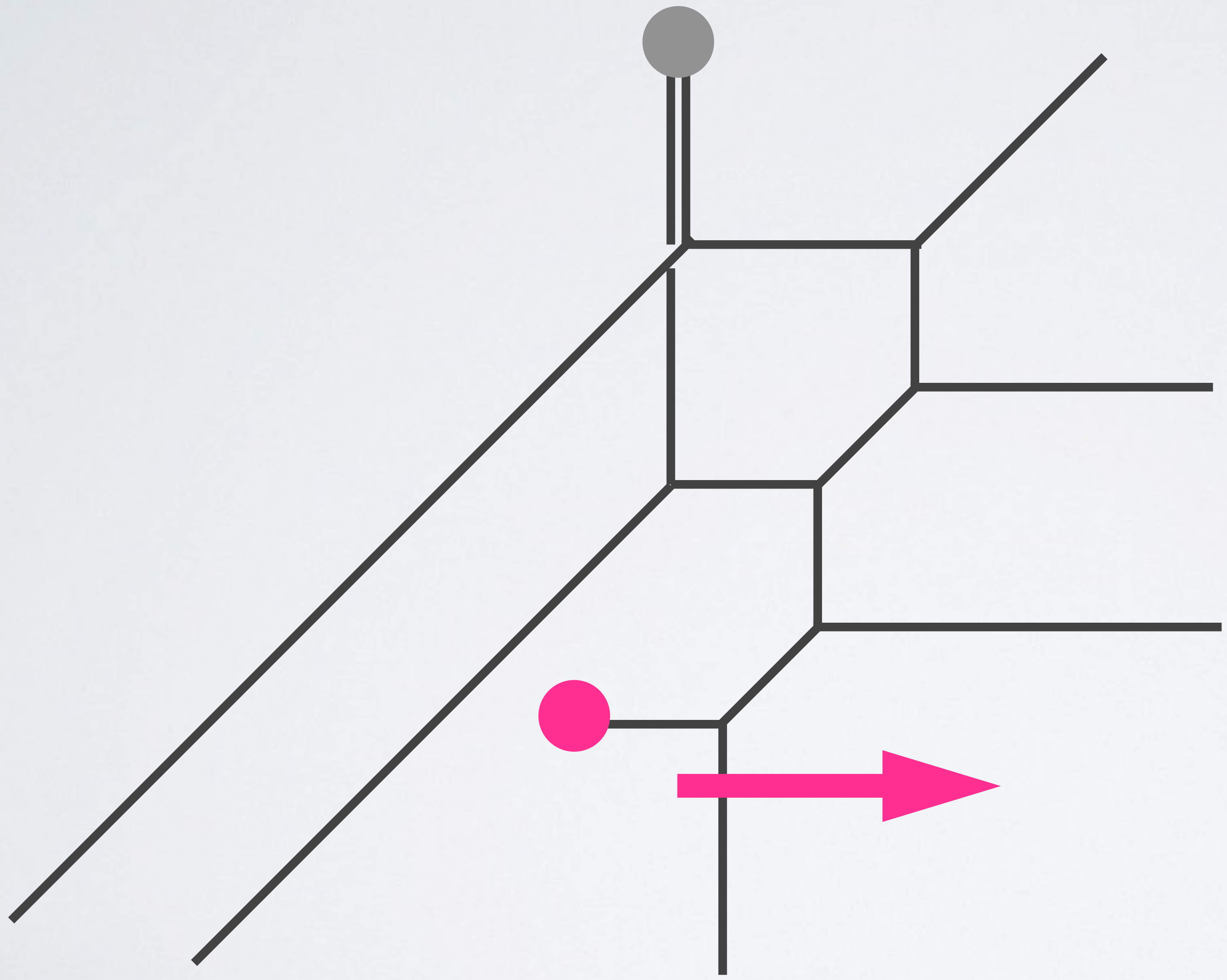

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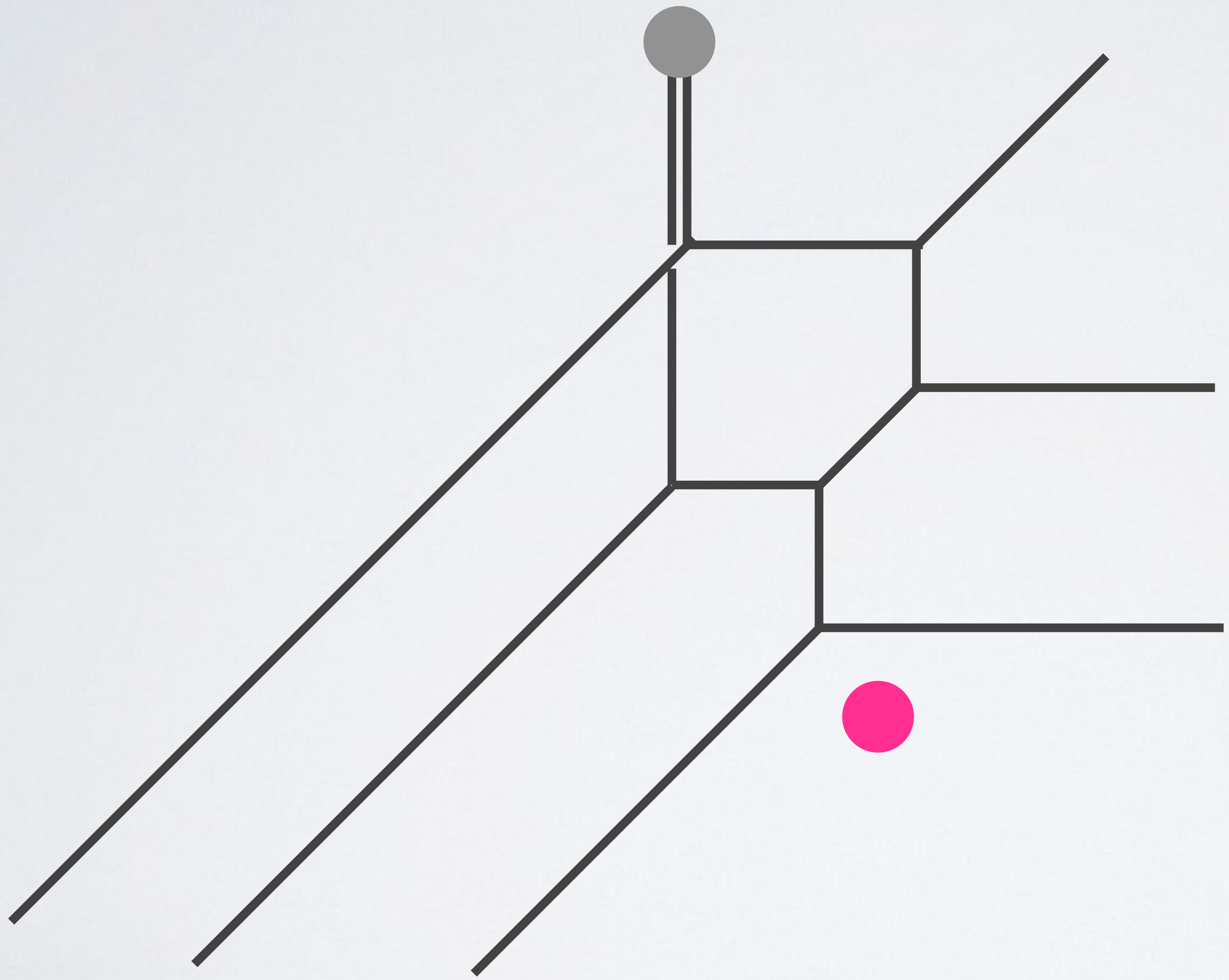

E-string case

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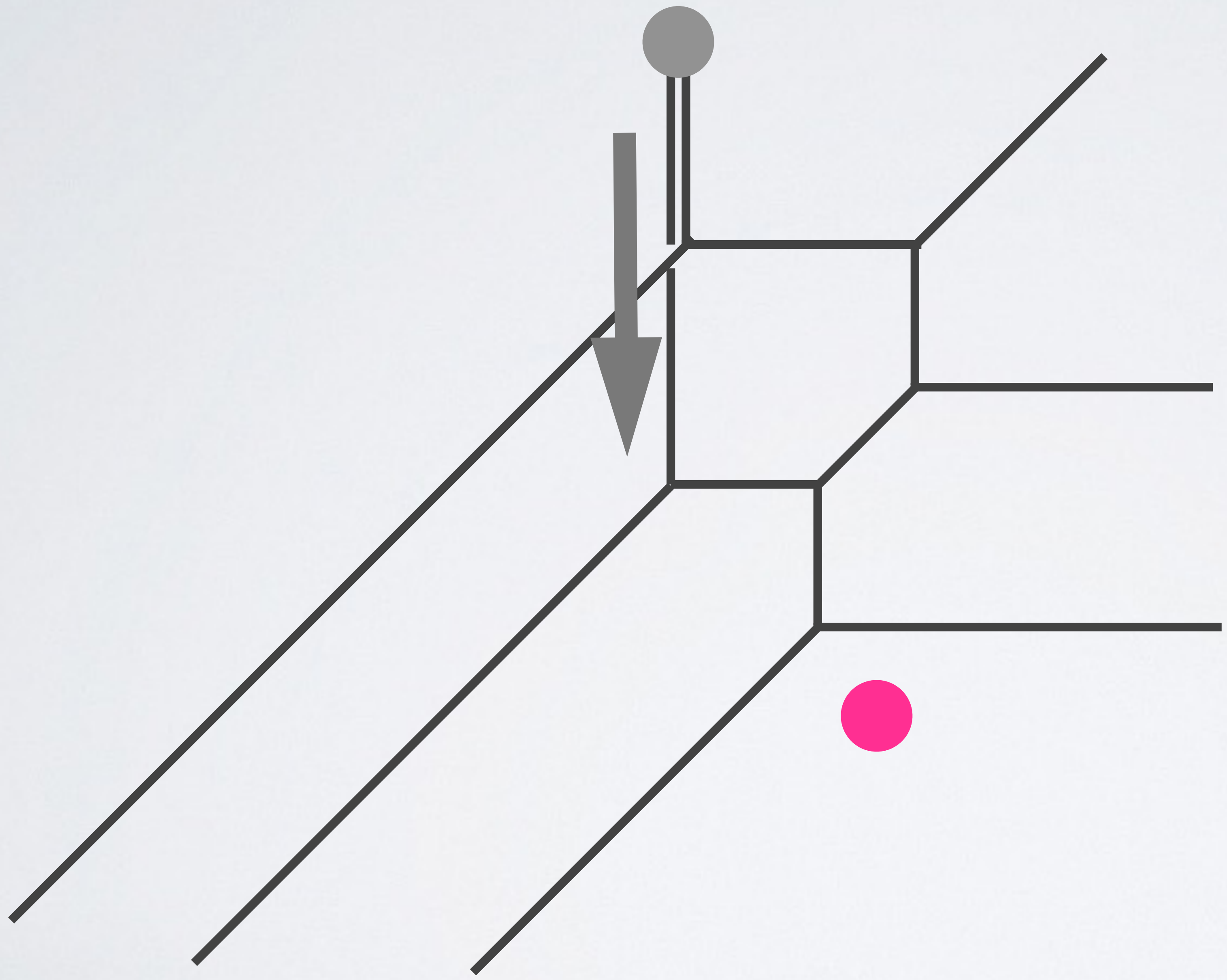

E-string case

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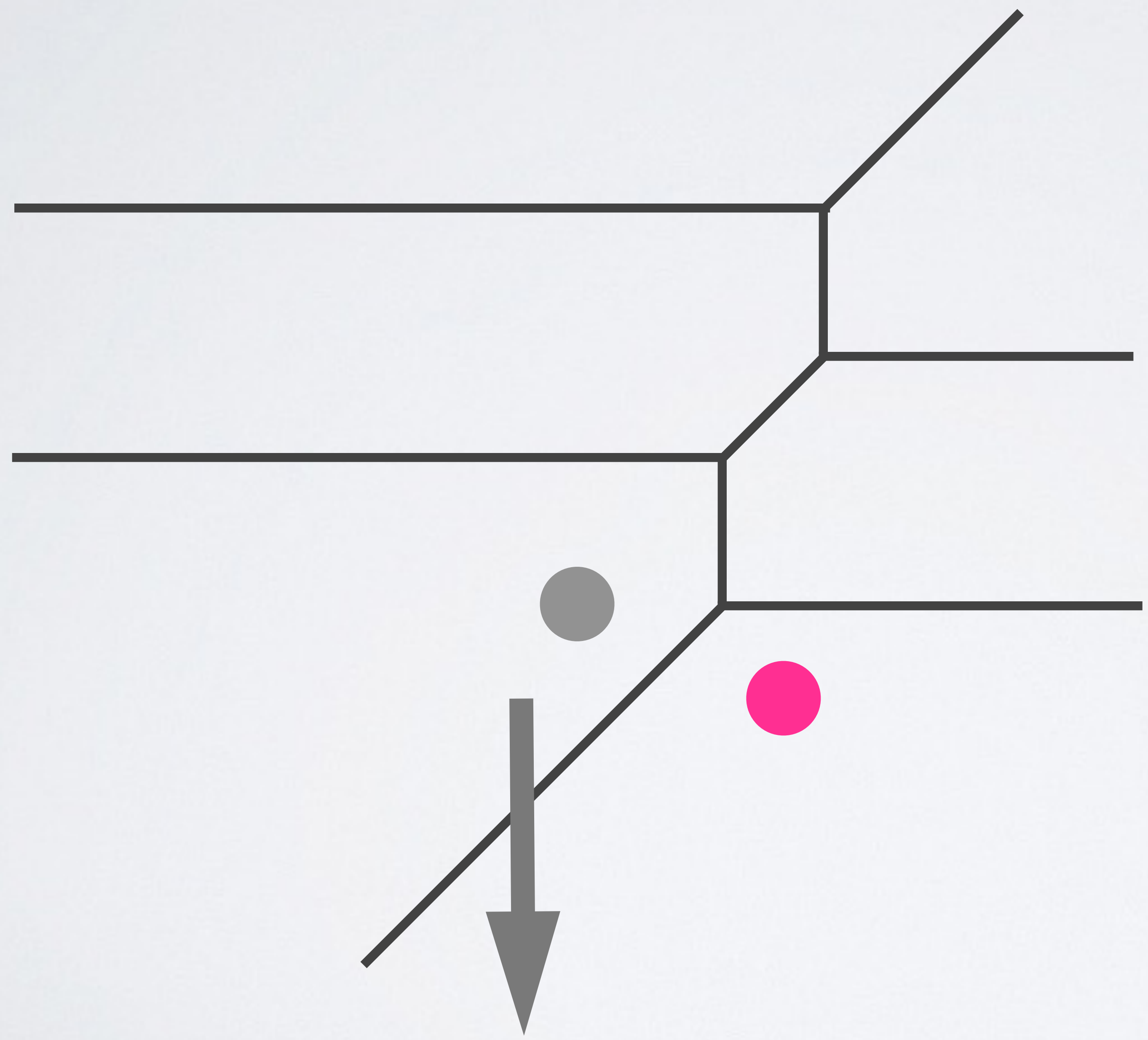

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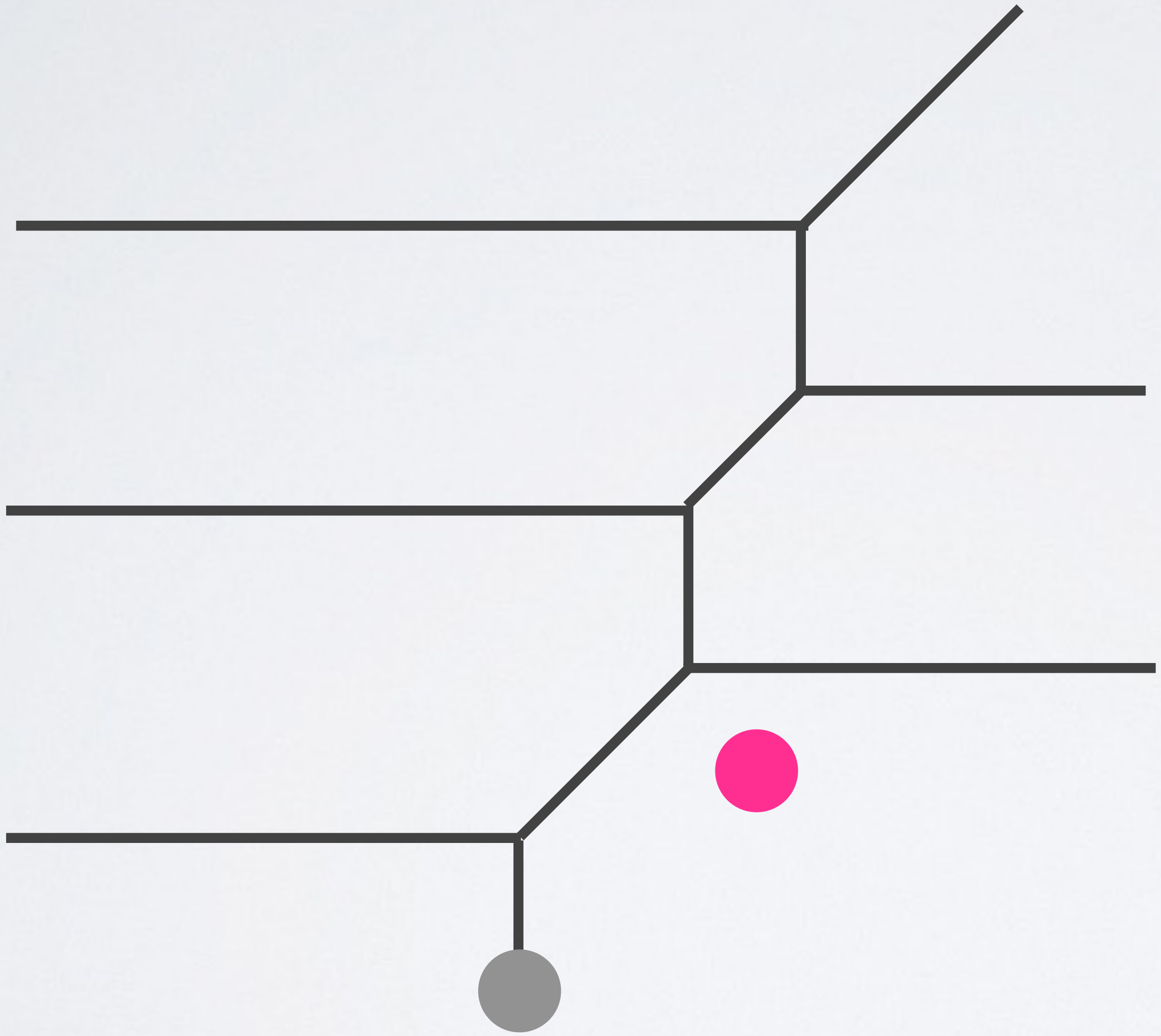

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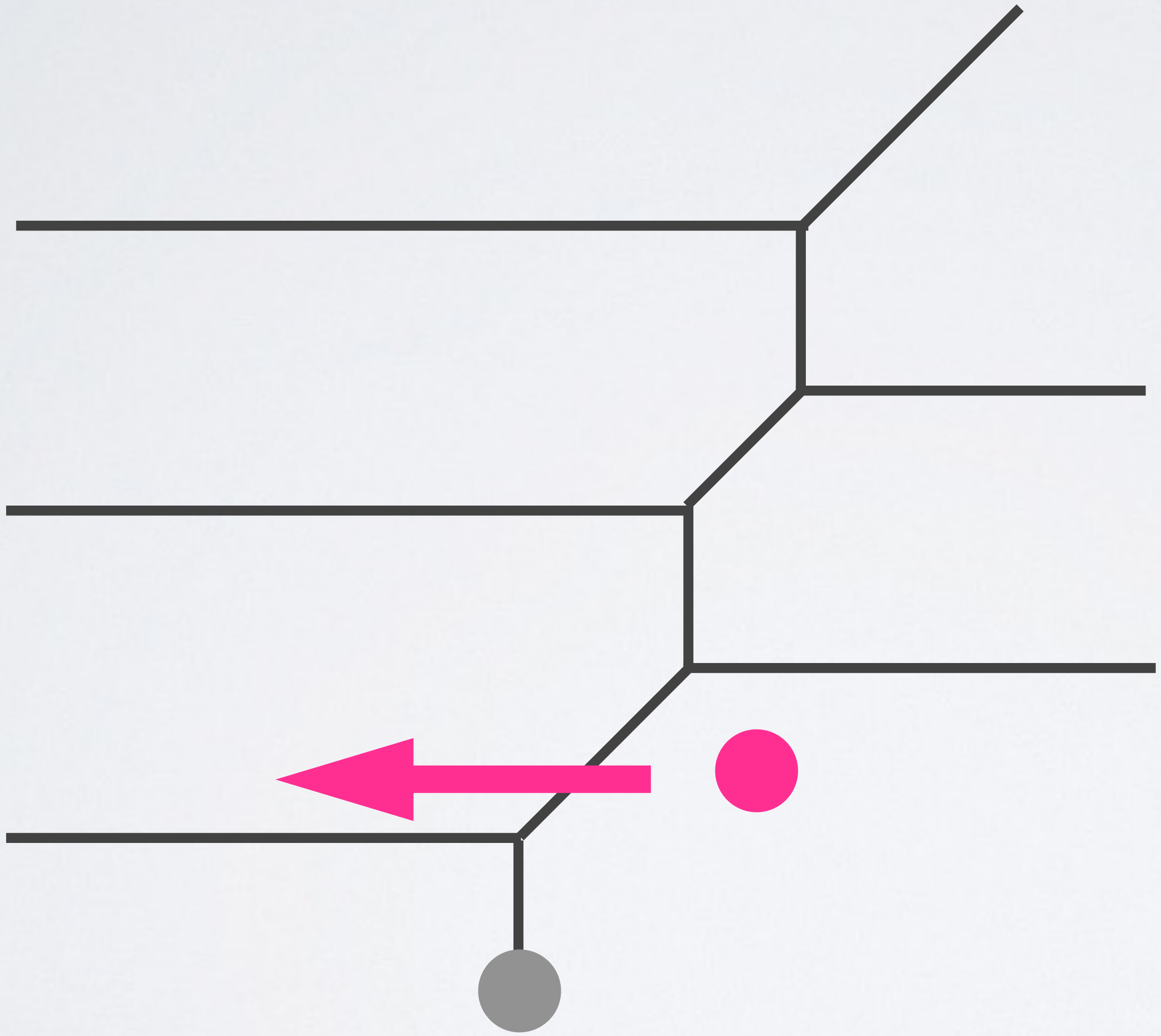

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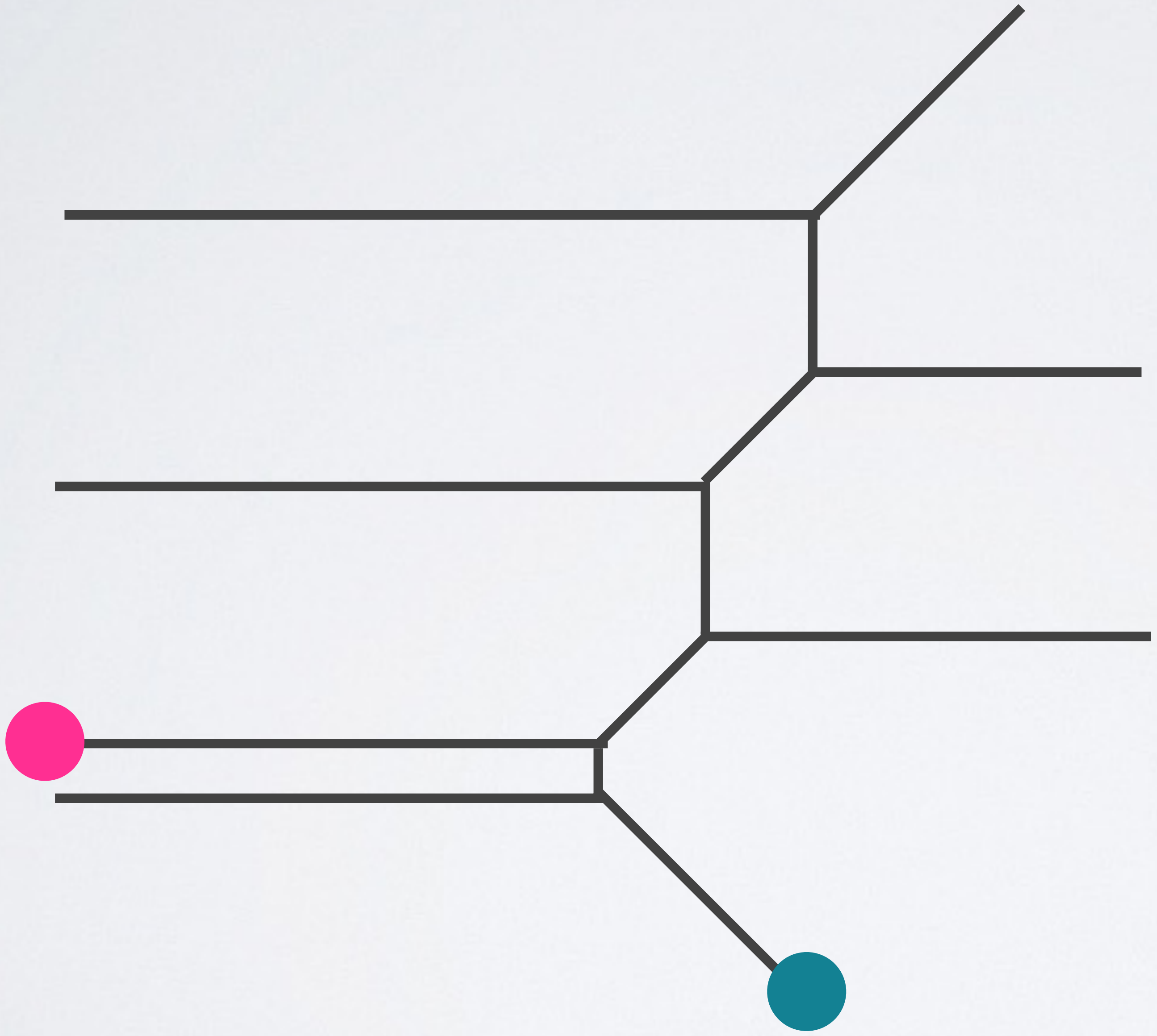
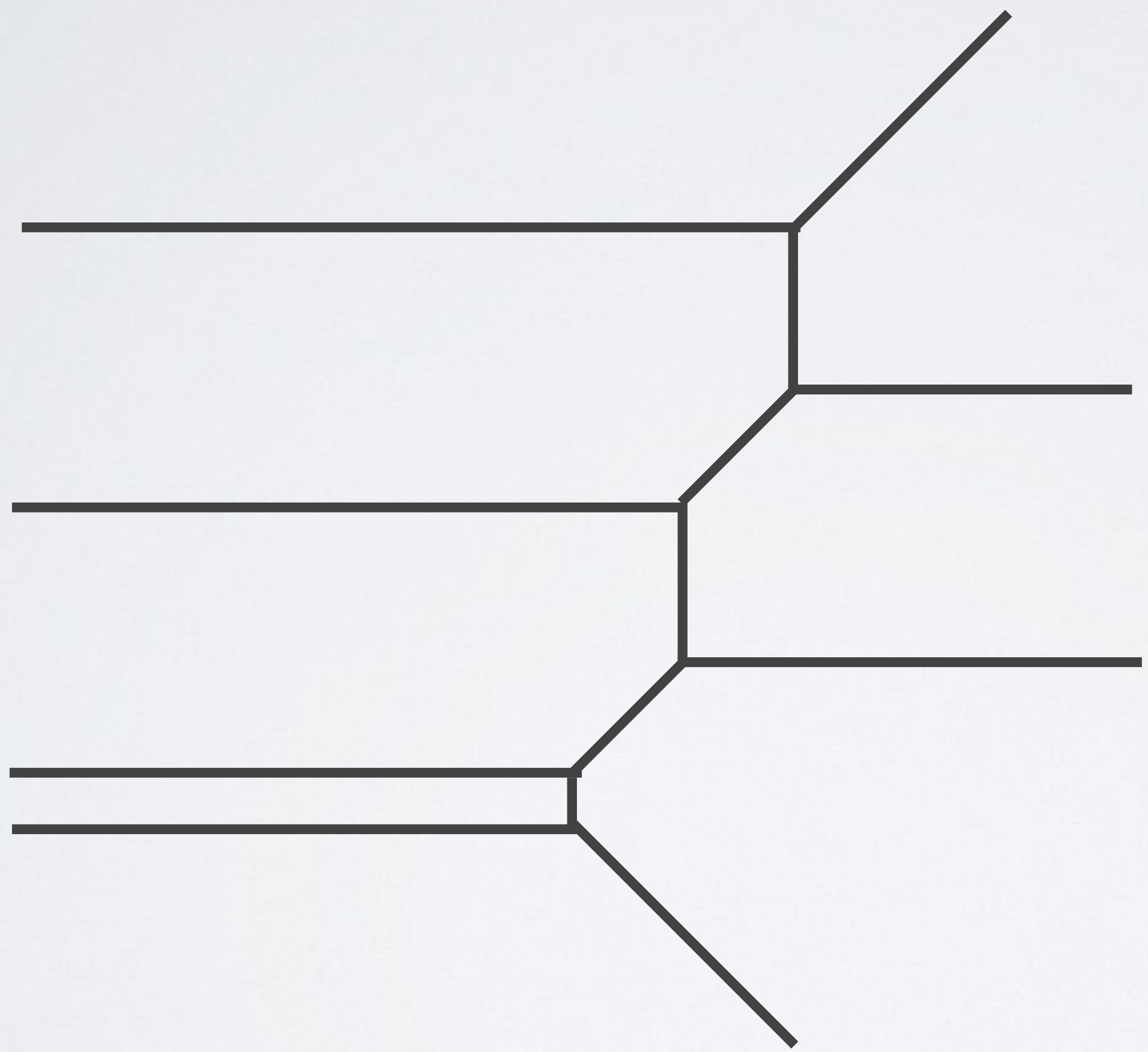


E-string case

We can compute this diagram exactly!



E-string case

Result (applied [Lockhart-Vafa, '12] after modifications)

$$Z^{S^5}_{pert} = Z_{class.} e^{\frac{\pi i}{6}\left(B_{3,3}(e^{2\pi i a}) + B_{3,3}(e^{2\pi i(a+\epsilon_1+\epsilon_2+1)})\right)}$$

$$\prod_{f=1}^{8} e^{-\frac{\pi i}{6}\left(B_{3,3}(e^{2\pi i(\frac{a}{2}+m_f+\frac{\epsilon_1+\epsilon_2+1}{2})}) + B_{3,3}(e^{2\pi i(-\frac{a}{2}+m_f+\frac{\epsilon_1+\epsilon_2+1}{2})})\right)}$$

$$S_3(e^{2\pi i a}) S_3(e^{2\pi i(a+\epsilon_1+\epsilon_2+1)}) S_3^{reg}(e^{2\pi i 0}) S_3(e^{2\pi i(0+\epsilon_1+\epsilon_2+1)})$$

$$\prod_{f=1}^{8} \frac{1}{S_3(e^{2\pi i(\frac{a}{2}+m_f+\frac{\epsilon_1+\epsilon_2+1}{2})}) S_3(e^{2\pi i(-\frac{a}{2}+m_f+\frac{\epsilon_1+\epsilon_2+1}{2})})}$$

E-string case

Result (applied [Lockhart-Vafa, '12] after modifications)

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I got 2 implications

Implication 1

Result (applied [Lockhart-Vafa, '12] after modifications)

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Implication 1

In order to make the first line consistent, the refinement of the classical part of topological string must be

$$\frac{1}{24}\Big(\frac{1}{\hbar^2}-1\Big)\int_{CY} J\wedge c_2$$

$$\to\ -\frac{1}{24}\Big(\frac{\epsilon_1}{\epsilon_2}+\frac{\epsilon_2}{\epsilon_1}+\frac{1}{\epsilon_1\epsilon_2}+3+3\frac{\epsilon_1+\epsilon_2}{\epsilon_1\epsilon_2}\Big)\int_{CY} J\wedge c_2$$

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Novel term (no such term in [Lockhart-Vafa, '12] etc)

Implication 2

masses are shifted by Omega-bkgd

$$Z_{pert}^{S^5} = Z_{class.} e^{\frac{\pi i}{6}\left(B_{3,3}(e^{2\pi i a}) + B_{3,3}(e^{2\pi i(a+\epsilon_1+\epsilon_2+1)})\right)}$$

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special shifted mass [Bobev-Bullimore-Kim, '15]

$$\tilde{m} = m + \frac{\epsilon_1 + \epsilon_2 + 1}{2}$$

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This comes from analytically-continuing the BPS state counting by topological strings to the expression based on triple trigonometric function

$$\prod_{i,j}(1 + zq^{i-1/2}t^{j-1/2})\cdots \to S_3\Big(-\sqrt{\frac{q}{t}}z\Big)$$

Open problem in instanton part

Anomaly polynomial computation

[Bobev-Bullimore-Kim, '15] 's prescription

$$\lim_{g^2\to\infty} -\log Z^{S^5} = \int A_{Estring} - \int A_{free\,tensor}$$

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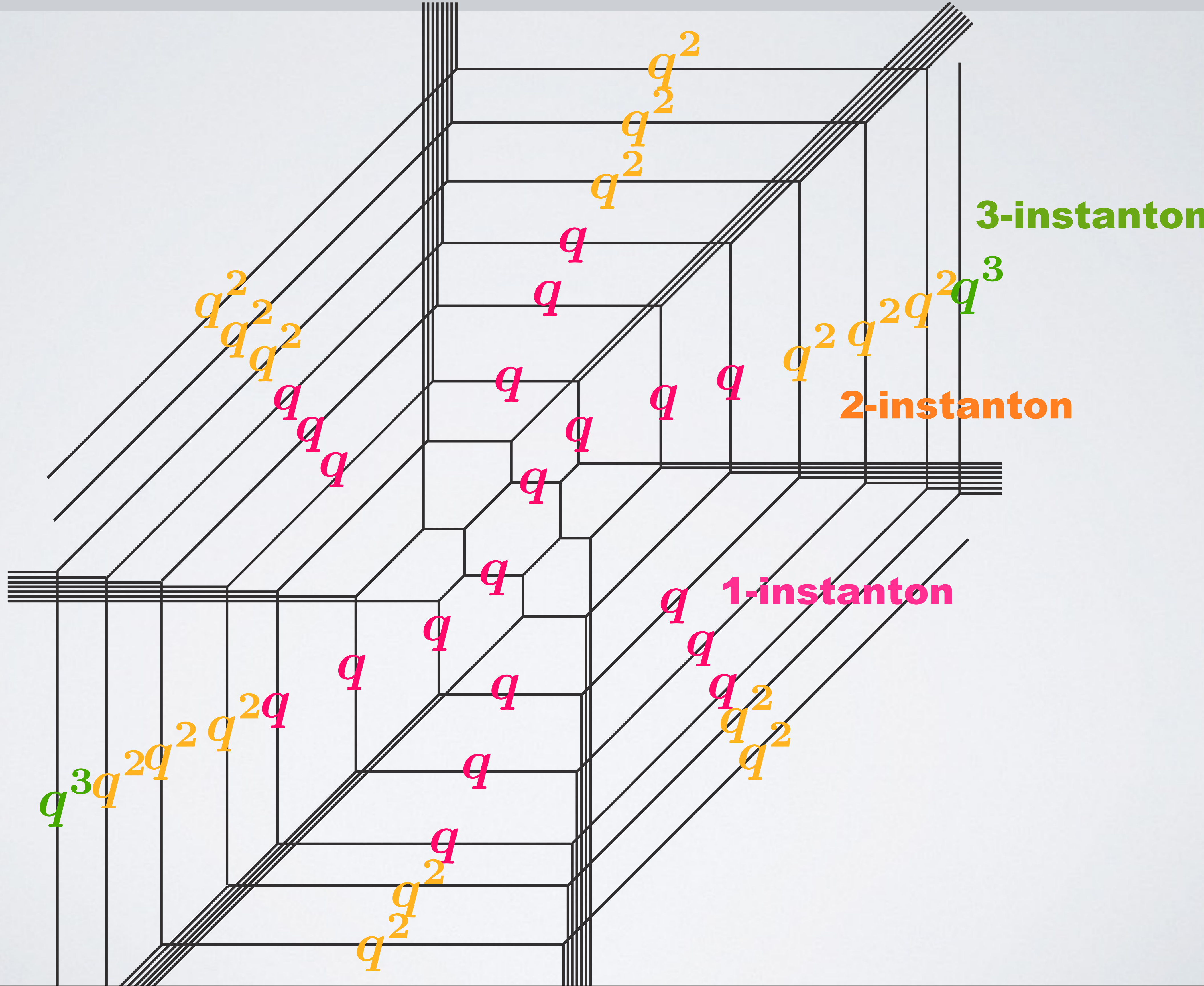
[Bobev-Bullimore-Kim, '15] 's prescription

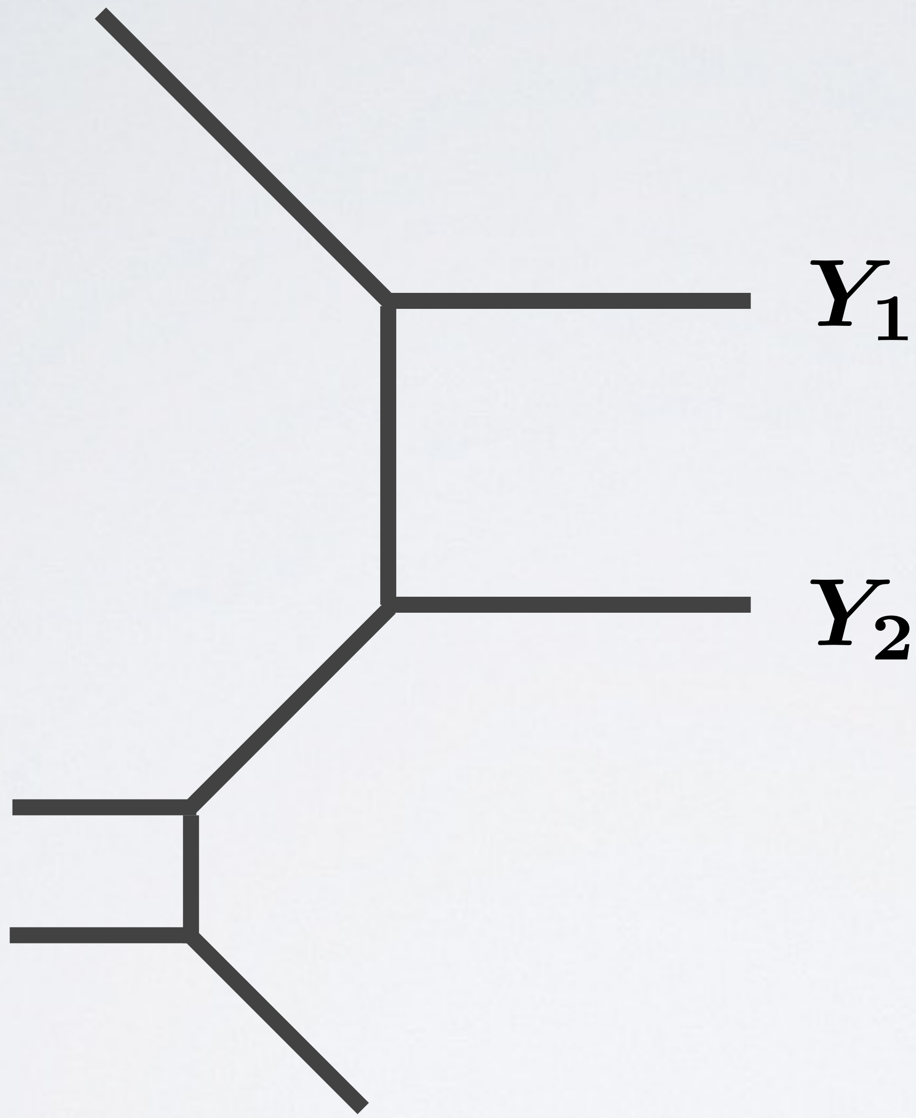
$$\lim_{g^2 \to \infty} -\log Z^{S^5} = \int A_{Estring} - \int A_{free\,tensor}$$

we need instanton correction for exact correspondence

Instanton computation is involved



7-brane tech. for instanton correction?



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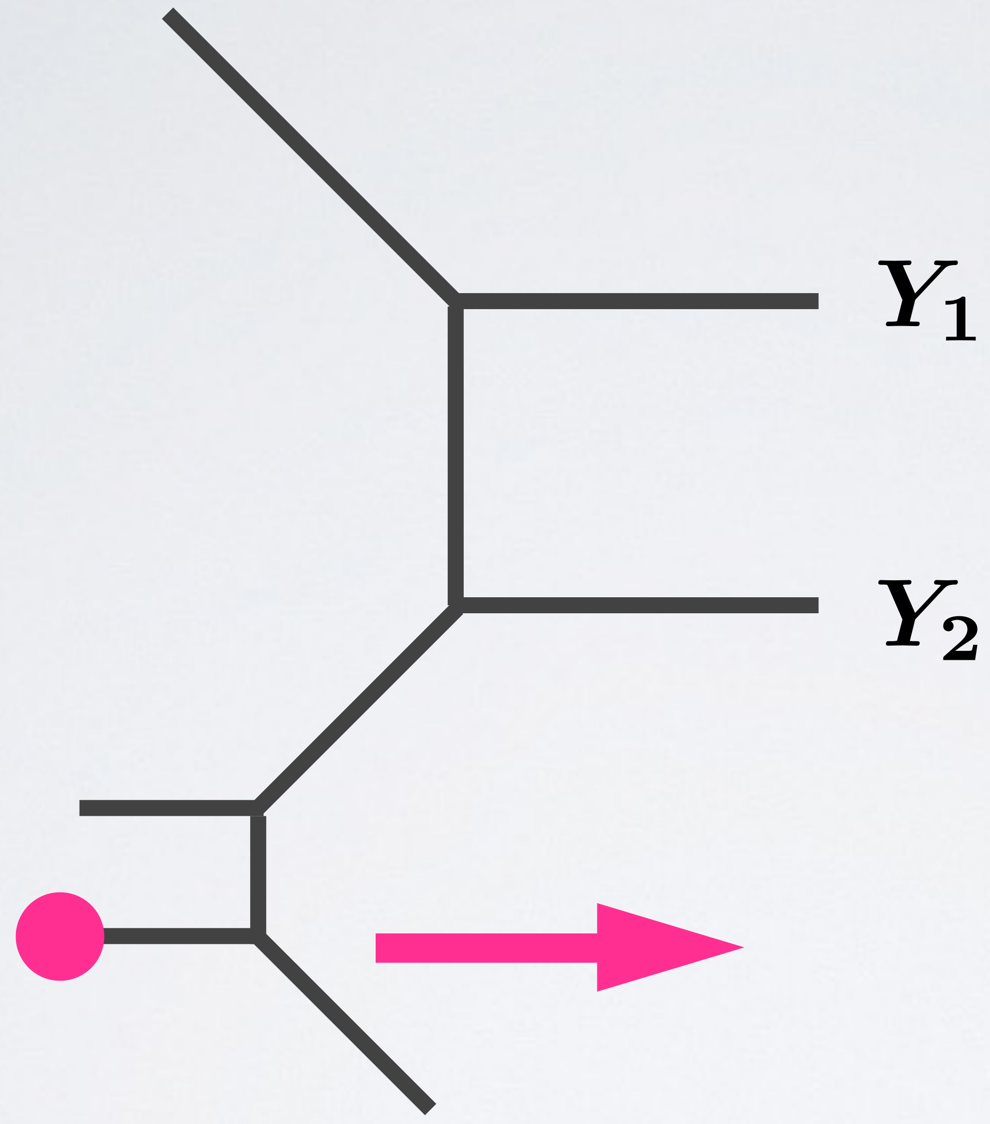

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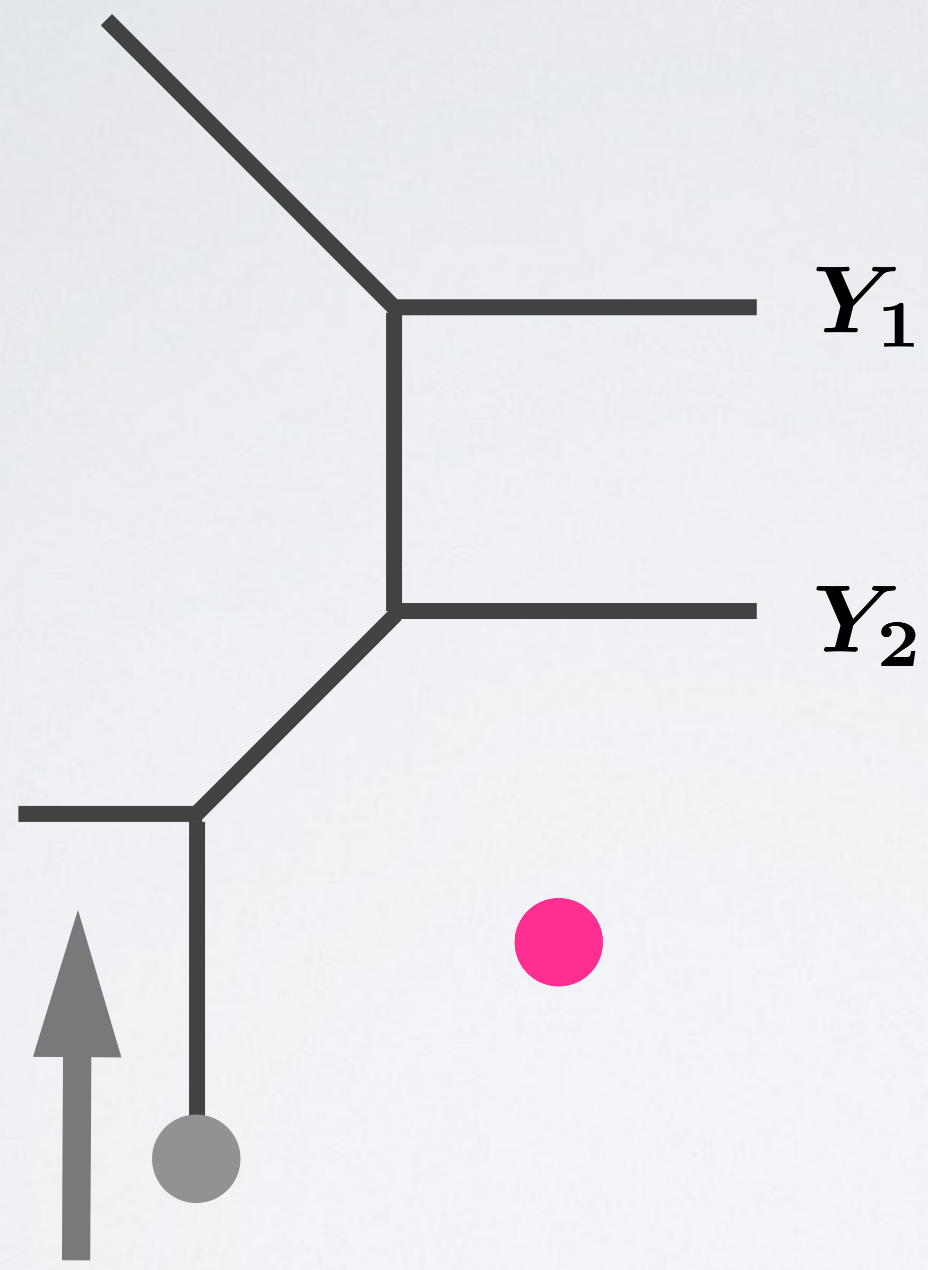

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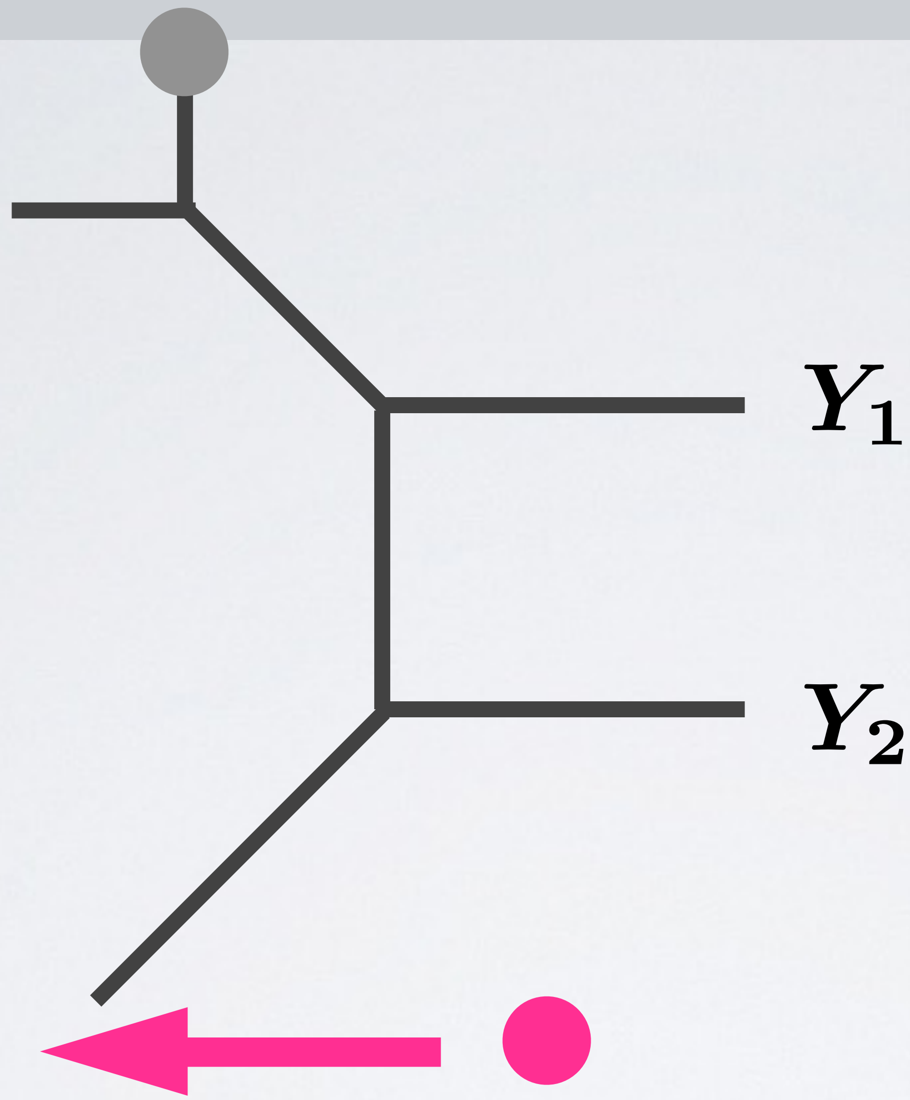

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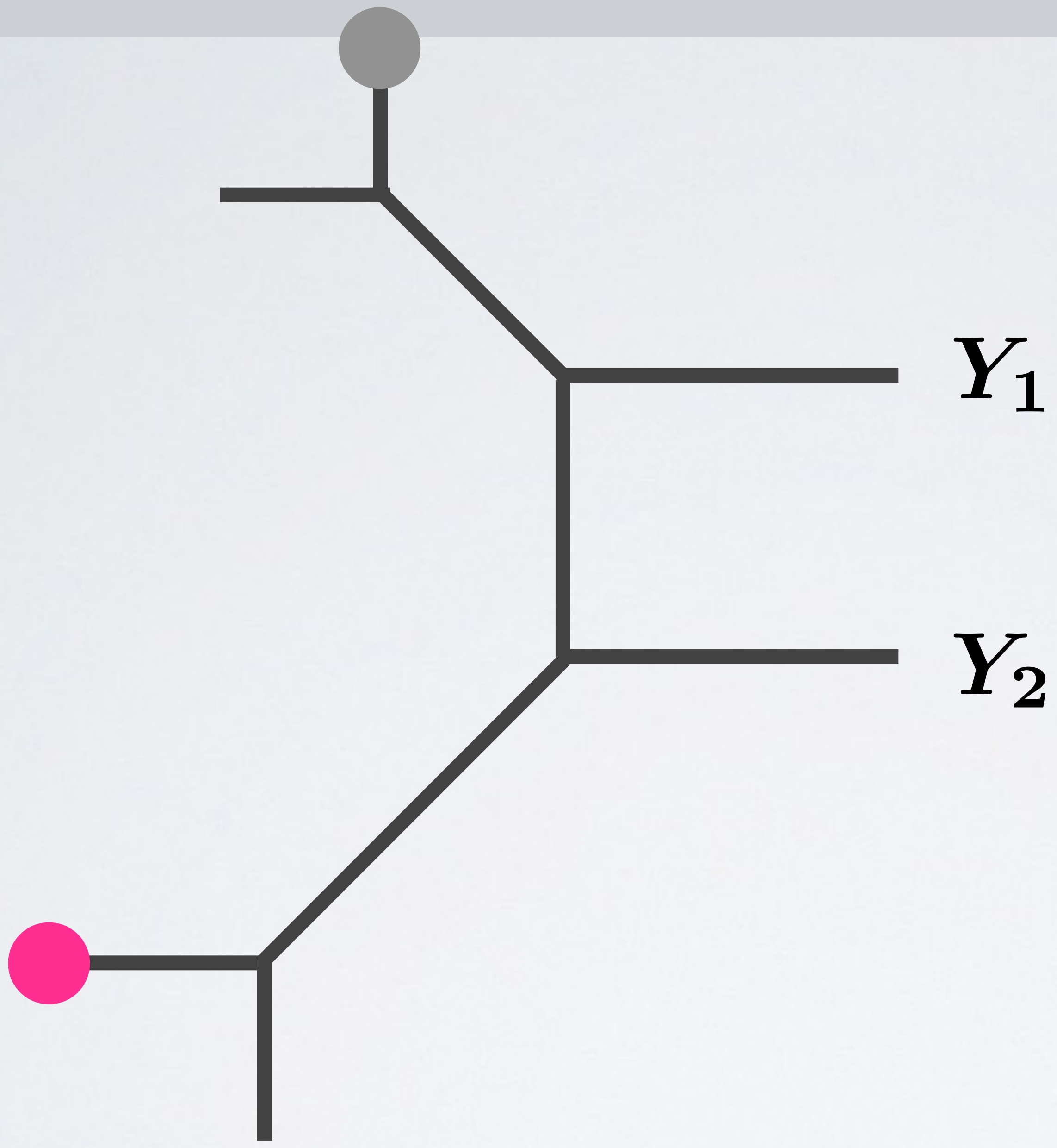

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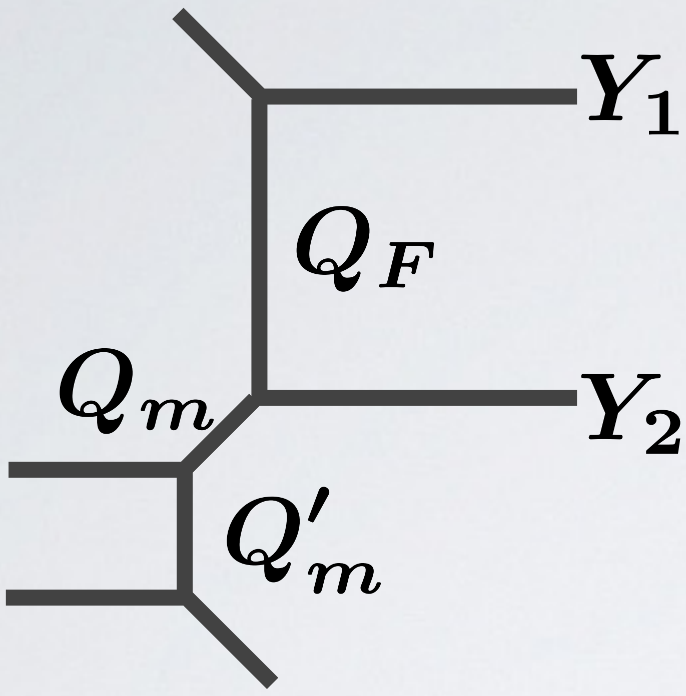
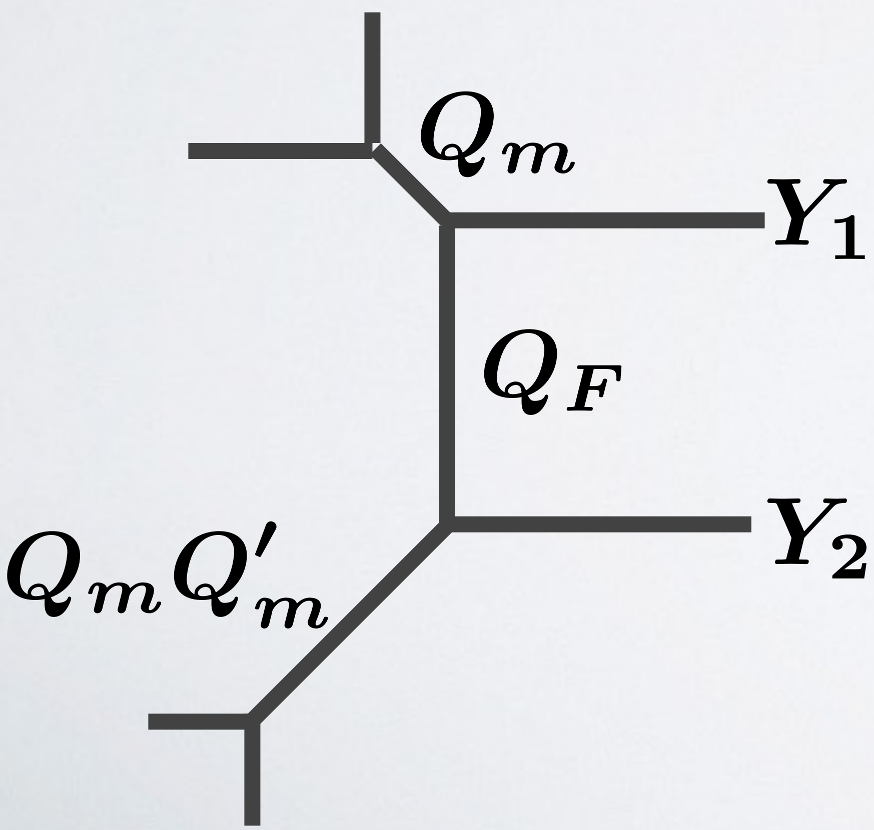
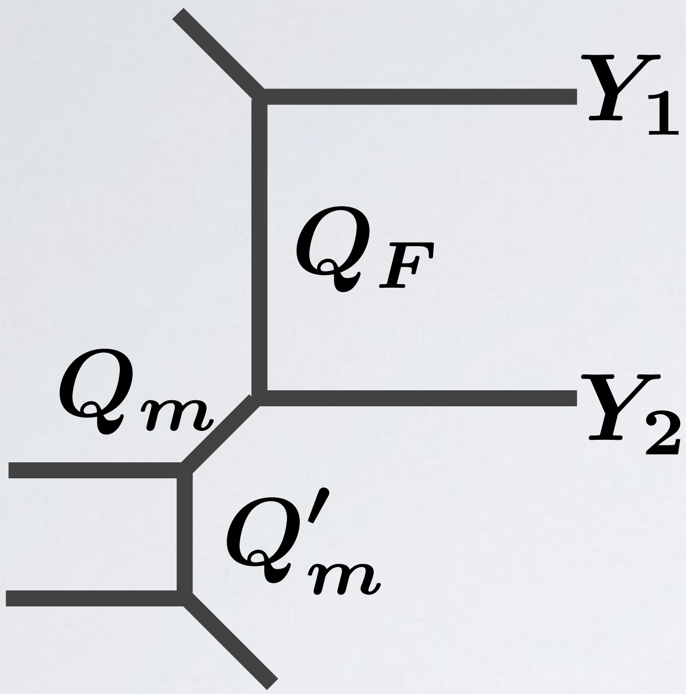


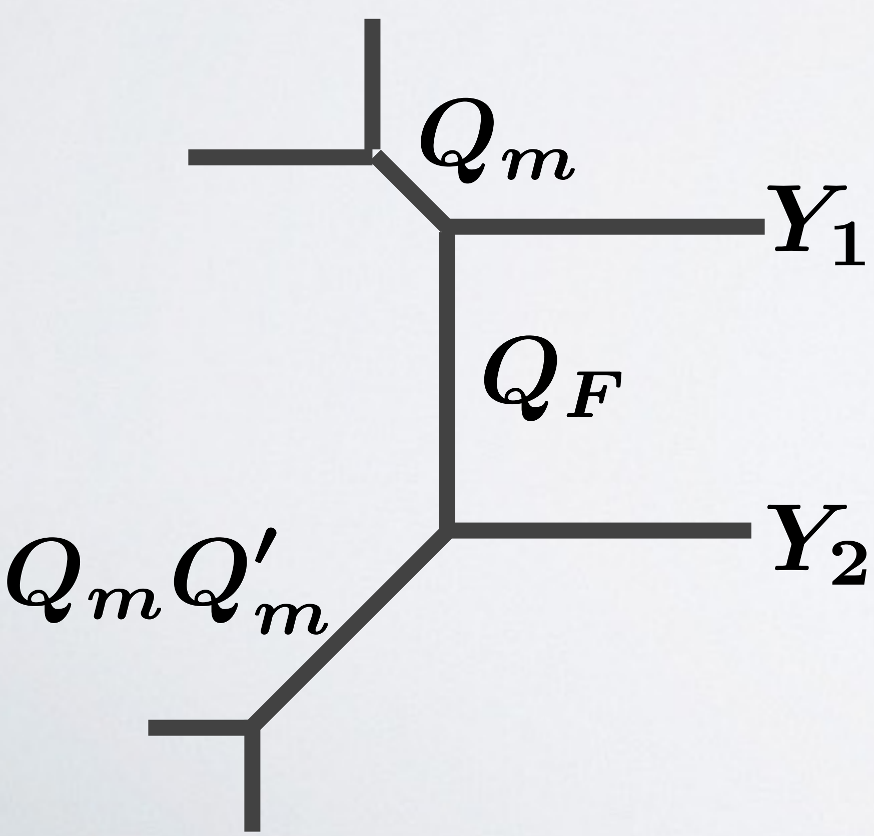


Open questions



$$\prod_{(i,j)\in Y_1}\left(1-Q_FQ_m^{-1}t^{-i+1/2}q^{j-1/2}\right)\prod_{(i,j)\in Y_2}\left(1-Q_m^{-1}t^{-i+1/2}q^{j-1/2}\right)$$

$$\neq$$



$$\prod_{(i,j)\in Y_1}\left(1-Q_m^{-1}t^{i-1/2}q^{-j+1/2}\right)\prod_{(i,j)\in Y_2}\left(1-Q_FQ_m^{-1}t^{i-1/2}q^{-j+1/2}\right)$$

Summary & open questions

topological strings -> anomaly polynomial by BBK

instanton correction is involved

we can check for various Tao webs

direct derivation from topological strings !?

Fin